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Welding Research Council, Inc.

Bulletin

Precision Equations and Enhanced Diagrams for Local Stresses in Spherical and Cylindrical Shells Due to External Loadings for Implementation of WRC Bulletin 107

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E²G | The Equity Engineering Group, Inc.

WRC – Welding Research Council, Inc. (WRC) brings together science and engineering specialists in developing the solutions to problems in welding and pressure vessel technology. They exchange knowledge, share perspectives, and execute R and D activities. As needed, the Council organizes and manages cooperative programs.

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FOREWORD

WRC Bulletin 537 is intended to facilitate implementation of the widely required and used relations found in the March 1979 revision of WRC Bulletin 107 for local stresses in spherical and cylindrical shells due to external loadings. The original analytical and experimental work of WRC's Pressure Vessel Research Council has become an essential tool for pressure vessel design for 45 years and in its present form for over 30 years. WRC Bulletin 537 has been prepared in response to numerous requests over the years for the precise equations depicted in the figures given in WRC Bulletin 107 (1979 Edition). The objective was to eliminate potential errors in implementation, facilitate proper interpolation and extrapolation and permit efficient computation with modern computers.

Mike Straub offered WRC his work at digitizing the numerous curves found in WRC Bulletin 107. David A. Osage organized the extensive effort to precisely capture the details of each curve in every single figure and develop the complex mathematical relations which render the new document useful for modern engineering practice. The effort and assuring the accuracy of the results required a great deal of time and attention to details. Involved in developing the equations and checking the results on Dave Osage's team were Mary Buchheim, David Amos, Tiffany Shaughnessy, and Debbie Samodell.

Welding Research Council, Inc. will no longer deliver WRC Bulletin 107 when requested for purchase. WRC Bulletin 537 provides the same content in a more useful and clearer format and is not an update or revision of WRC Bulletin 107. Instead, it is the 2010 printing of WRC Bulletin 107 and has been meticulously checked. Those responsible for codes, standards and specifications that require use of WRC Bulletin 107 should amend those documents to reflect the fact that WRC Bulletin 537 is the equivalent to WRC Bulletin 107 and provides the same acceptable basis for design.

Since the 2010 Edition of WRC Bulletin 537 three Errata's have been issued. The third Errata issued on August 19, 2011 was accumulative and comprised of the first and second Errata's issued on, respectively, April 2011 and June 27, 2011.

WRC Bulletin 537 2nd Edition, published in October 2020, replaces previous versions of WRC Bulletin 537 as well as the first, second and third Errata. WRC Bulletin 537 2nd Edition is all-inclusive; changes have been made to incorporate each item identified in the first, second and third Errata; they are:

- Correct range for beta from 0.375 to 0.5 in the August 6, 1965 Foreword.
- Correct page number to 43 for nondimensional curves for spherical shells.
- Correct page number to 91 for nondimensional curves for cylindrical shells should.
- Computation of Nø/(P/Rm) currently states 3C or 4C, should only State 3C in Table 5. (Note: Figure 3C is properly labeled.)
- Computation of Nx/(P/Rm) currently states 3C or 4C, should only State 4C in Table 5. (Note: Figure 4C is properly labeled.)
- Incorrect column name in last column, currently shows My should show Ny in the Curve Fit Table for SPI-1.
- Curve fit for data was adjusted to prevent negative values for Y within valid range for X in Figure SM-3

and Curve Fit Table.

- Adjusted the placement of the labels for Nx and My for easier reading in Figure SM-9.
- Equation shown is not the equation used to calculate the curve fit in the Curve Fit Table for SM-10.
- Missing figure and table for Figure 4A and Curve Fit Table.
- Figure 3B Title is incorrect and should be longitudinal moment not circumferential.
- Figure 4B Title is incorrect should be longitudinal moment not circumferential.
- Figure 1C-1 Original and Curve Fit Table display incorrect range for Y.
- Figure 1C-1 Extrapolated and Curve Fit Table display incorrect range for Y.
- Figure B-2 has incorrect values for x axis.

Additional changes made to WRC Bulletin 537 2nd Edition are as follows:

- Changed WRC Bulletin number from 537 to 537 2nd Edition.
- Corrected header throughout Bulletin.
- Changed the cross reference from a specific page number to Section 8 for nondimensional curves for spherical shells in Section 3.4.
- Changed the cross reference from a specific page number to “clicking here” for nondimensional curves for cylindrical shells in Section 4.4.1.
- Changed in-line MathType text to Microsoft’s LaTeX Math Equation syntax.

Dr. Martin Prager

Executive Director, WRC

FOREWORD

To WRC Bulletin 537, 2011 Edition

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Mike Straub offered WRC his work at digitizing the numerous curves found in WRC 107. Mr. David A. Osage organized an extensive effort to precisely capture the details of each curve in each figure and develop the complex mathematical relations which render the new document useful for modern engineering practice. The effort and assuring the accuracy of the results required a great deal of time and attention to details. Involved in developing the equations and checking the results on Dave Osage's team were Mary Buchheim, David Amos, Tiffany Shaughnessy, and Debbie Samodell.

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Since the 2010 edition of WRC Bulletin 537 three Errata's have been issued. The first Errata was issued in April 2011, the second Errata was issued on June 27, 2011 and the third Errata issued on August 19, 2011. Third Errata issued in August of 2011 was accumulative.

Dr. Martin Prager

Executive Director, WRC

FOREWORD

To WRC Bulletin 537, 2010 Edition

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Dr. Martin Prager

Executive Director, WRC

FOREWORD

To WRC Bulletin 107, October 2002 Update of March 1979 Revision

The October 2002 Update to the March 1979 Revision of WRC Bulletin 107 includes minor editorial changes for improvement and readability of several equations, curves, and some text. There are NO technical changes.

- The calculation forms (Tables 2, 3 and 5) are improved, particularly to show the "+" and "-" quantities more definitively.
- The equation for stress in paragraph 3.6.3 is revised to be on one line.
- The parameter definitions on several of the curves (beginning with Figure SR-1) are improved and clarified.
- Appendix B, exponents in Equations 1, 2, 3 and 4 are enlarged for readability.

PVRC thanks Mr. James R. Farr, Honorary Emeritus Member of the Pressure Vessel Research Council, for his assistance in preparing this update.

NOTE: WRC Bulletins 107 and 297 should be considered (and purchased) as an integral set. In addition, PVRC Technical Committees are working on a project that is envisioned to culminate in a new publication to add to the WRC Bulletin 107 and 297 set. The new publication will provide significant new technical information on local shell stresses from nozzles and attachments.

Greg L. Hollinger

The Pressure Vessel Research Council

FOREWORD

To WRC Bulletin 107, March 1979 Update of August 1965 Original Version

Welding Research Council, Inc. Bulletin No. 107 has been one of the most widely used bulletins ever published by WRC. The original bulletin was published in August 1965. Since that time, a revised printing was issued in December 1968; a second revised printing was issued in July 1970; a third revised printing was released in April 1972; and a June 1977 reprint of the third revised printing was issued. As sometimes happens with publications of this type, some errors were detected and then corrected in subsequent revised printings.

In this March 1979 Revision of Bulletin 107, there are some additional revisions and clarifications. The formulations for calculation of the combined stress intensity, S, in Tables 2, 3, and 5 have been clarified. Changes in labels in Figures 1C-1, 2C-1, 3C, and 4C have been made and the calculated stresses for Model "R" in Table A-3 and Model "C-I" in Table A-4 have been revised accordingly. The background for the change in labels is given in a footnote on page 66.

Present plans call for a review and possible extension of curves to parameters which will cover most openings in nuclear containment vessels and large storage tanks. Plans are to extend R/T from 300 to 600 and to extend d/D range from 0.003 to 0.10 for the new R/T range, review available test data to establish limits of applicability, and develop some guidance for pad reinforcements.

Long range plans are to review shell theory in general, and Bijlaard's method in particular. The goal is to extend the R/T up to 1200 for a d/D up to 0.1. This will include large deflection theory and other nonlinear effects. In addition, available computer programs will be studied in hope of developing one which will be an appropriate supplement to Bijlaard's method. Finally, a review will be made of limit loads related to large R/T and small d/D.

J.R. Farr, Chairman

PVRC Design Division

FOREWORD

To WRC Bulletin 107, August 1965 Original Version

Several years ago, the Pressure Vessel Research Committee sponsored an analytical and experimental research program aimed at providing methods of determining the stresses in pressure vessel nozzle connections subjected to various forms of external loading. The analytical portion of this work was accomplished by Prof. P.P. Bijlaard of Cornell University, and was reported in references [1] to [8] inclusive. Development of the theoretical solutions involved several simplifying assumptions, including the use of shallow shell theory for spherical vessels and flexible loading surfaces for cylindrical vessels. These circumstances limited the potential usefulness of the results to d_i/D_i, ratios of perhaps 0.33 in the case of spherical shells and 0.25 in the case of cylindrical shells. Since no data were available for the larger diameter ratios, Prof. Bijlaard later supplied data, at the urging of the design engineers, for the values of $\beta = 0.375$ and 0.50 (d_i/D_i, ratios approaching 0.60) for cylindrical shells, as listed on page 12 of reference [10]. In so doing, Prof. Bijlaard included a specific warning concerning the possible limitations of these data, as follows: "The values for these large loading surfaces were computed on request of several companies. It should be remembered, however, that they apply to flexible loading surfaces and, for radial load, to the center of the loading surface. It should be understood that using these values for the edge of the attachment, as was recommended for small loading surfaces, may be unconservative."

Following completion of the theoretical work, experimental work was undertaken to verify the theory, the results of which were published in references [17] and [18]. Whereas this work seemingly provided reasonable verification of the theory, it was limited to relatively small d_i/D_i ratios-0.10 in the case of spherical shells and 0.126 in the case of cylindrical shells. Since virtually no data, either analytical or experimental, were available covering the larger diameter ratios, the Bureau of Ships sponsored a limited investigation of this problem in spheres, aimed at a particular design problem, and the Pressure Vessel Research Committee undertook a somewhat similar investigation in cylinders. Results of this work have recently become available emphasizing the limitations in Bijlaard's data on cylindrical shells, particularly as it applies to thin shells over the "extended range" (page 12 of reference [10]).

Incident to the use of Bijlaard's data for design purposes, it has become apparent that design engineers sometimes have difficulty in interpreting or properly applying this work. As a result of such experience, PVRC has felt it desirable that all Bijlaard's work be summarized in convenient, "cook-book" form to facilitate its use by design engineers. However, before this document could be issued, the above mentioned limitations became apparent, presenting an unfortunate dilemma, viz., the test data indicate that the calculated data are partially inadequate, but the exact nature and magnitude of the error is not known, nor is any better analytical treatment of the problem available (for cylinders).

Under these circumstances, it was decided that the best course was to proceed with issuing the "cook-book," extending Bijlaard's curves as best we can base on available test data. This decision was based on the premise that all of the proposed changes would be toward the conservative (or "safe") side and that design engineers would continue to use Bijlaard's extended range data unless some alternative were offered. The following paper is therefore presented in the hope that it will facilitate the use of Bijlaard's