BULLETIN 254 NOVEMBER 1979

ISSN 0043-2326

Welding Research Council bulletin

A CRITICAL EVALUATION OF PLASTIC BEHAVIOR DATA AND A UNIFIED DEFINITION OF PLASTIC LOADS FOR PRESSURE COMPONENTS

J. C. Gerdeen

INTERPRETIVE REPORT ON LIMIT ANALYSIS AND PLASTIC BEHAVIOR OF PIPING PRODUCTS

E. C. Rodabaugh

INTERPRETIVE REPORT ON LIMIT ANALYSIS OF FLAT CIRCULAR PLATES

W. J. O'Donnell

These Bulletins contain final Reports from projects sponsored by the Welding Research Council, important papers presented before engineering societies and other reports of current interest.

Bulletin No. 254/$13.50 per copy

WELDING RESEARCH COUNCIL
UNITED ENGINEERING CENTER
345 EAST 47th STREET, NEW YORK, N.Y. 10017

INTENTIONALLY LEFT BLANK

WRC - The Welding Research Council brings together science and engineering specialists in developing the solutions to problems in welding and pressure vessel technology. They exchange knowledge, share perspectives, and execute R and D activities. As needed, the Council organizes and manages cooperative programs.

MPC – A Council of the WRC, the Materials Properties Council is dedicated to providing industry with the best technology and the best data that can be obtained on the properties of materials to help meet today's most advanced concepts in design and service, life assessment, fitness-for-service, and reliability and safety.

PVRC – A Council of the WRC, the goal of the Pressure Vessel Research Council is to encourage, promote and conduct research in the field of pressure vessels and related pressure equipment technologies, including evaluation of materials, design, fabrication, inspection and testing.

For more information, see www.forengineers.org

WRC Bulletins contain final reports from projects sponsored by the Welding Research Council, important papers presented before engineering societies and other reports of current interest.

Printed in the United States of America.

ISSN 0043-2326
Library of Congress Catalog Number: 85-647116

Welding Research Council
20600 Chagrin Blvd.
Suite 1200
Shaker Heights, OH 44122
www.forengineers.org

INTENTIONALLY LEFT BLANK

A Critical Evaluation of Plastic Behavior Data and A United Definition of Plastic Loads for Pressure Components

by J. C. Gerdeen

Foreward

For over two decades, the various subcommittees under the Design Division have been carrying out analytical and experimental research into the plastic behavior of pressure components consisting of pressure vessel heads, cylindrical piping, curved piping and elbows, nozzles in spherical vessels, nozzles in cylindrical shells, and flat circular plates. The analytical methods developed to quantify the plastic strength were primarily based on the concept of limit analyses which is strictly applicable to idealized elastic/perfectly plastic materials. Due to the obvious differences between the ideal and actual material behavior, several different methods were used to determine the plastic strength in experimental investigations. Discussions among the members of ASME Boiler and Pressure Vessel Code as well as among the active investigators indicated that there is a considerable amount of controversy about the basis and applicability of these methods. Since plastic strengths, determined by the methods of limit analysis as well as experimental procedures, have been used in the ASME Boiler and Pressure Vessel Code as an alternative basis for setting allowable limits on primary loadings, the Design Division of the PVRC felt it necessary to resolve this controversy. Consequently, the Task Group on Characterization of Plastic Behavior of Structures was set up in 1975.

Briefly, the objective of the Task Group was to critically review plastic behavior data and information, obtained under various PVRC Subcommittees as well as by other sources, to establish difinitions of limit and plastic collapse loads, and, finally, to recommend uniform procedures and standards for determining limit and plastic collapse loads for use in design criteria.

As a first step, the Task Group engaged Dr. J. C. Gerdeen, Mr. E. C. Rodabaugh, and Dr. W. J. O'Donnell to prepare the following three reports which summarized the plastic behavior data and relevent information obtained by various PVRC subcommittees and other sources:

1. J. C. Gerdeen, Summary and Interpretive Report on Limit Analysis, Elastic-Plastic Analysis, and Experiments on Shells, Michigan Technological University Report, April 1976.
2. E. C. Rodabaugh, Interpretive Report on Limit Analysis and Plastic Behavior of Piping Products, Battelle-Columbus Laboratories Report N-0584, October 1976.
3. W. J. O'Donnell, Interpretive Report on Limit Analysis of Flat Circular Plates, O'Donnell and Associates, Inc.

As a second and final step, the Task Group engaged Dr. J. C. Gerdeen to critically review the data contained in these three summary reports and to prepare a final report addressing the specific objectives of the Task Group. This report is entitled "A Critical Evaluation of Plastic Behavior Data and a Unified Definition of Plastic Loads for Pressure Components." This final report is published as the first article in this bulletin. The Task Group found that all of the relevent information from Dr. J. C. Gerdeen's summary report was included in the final report whereas a substantial amount of information from the summary reports of Mr. E. C. Rodabaugh and Dr. W. J. O'Donnell was not directly included in the final report and that the members of the Task Group felt that this information is useful for a complete understanding of the basis for reaching the conclusions presented in the final report. For this reason, the summary reports by Mr. E. C. Rodabaugh and Dr. W. J. O'Donnell are published as second and third article, respectively, in this bulletin.

Having completed its assignment, the Task Group was dissolved in May 1979. It is acknowledged that the Task Group was able to achieve its goals within a short period of four years due, primarily, to the excellent cooperation and active participation of the following members of the task group and the support provided by their respective organizations:

S. Palusamy, *Chairman,* Westinghouse Nuclear Technology Division
J. C. Gerdeen, *Engineering Consultant*
W. L. Greenstreet, Oak Ridge National Laboratories
G. F. Leon, General Dynamics Corp.
A. Lohmeier, Sumitomo Corp. of America
R. H. Mallett, Westinghouse Advanced Reactor Division
R. L. Maxwell, University of Tennessee, Secretary of the Task Group
W. J. O'Donnell, O'Donnell and Associates
P. R. Raju, Teledyne Engineering Services
M. Ramchandani, Burns & Roe, Inc.
E. C. Rodabaugh, Battelle-Columbus Laboratories

Sam S. Palusamy, Chairman

PVRC Task Group on Characterization of Plastic Behavior of Structures

J. C. Gerdeen is an Engineering Consultant and is located in Minneapolis, MN.

Publication of this report was sponsored by the **Task Group on the Characterization of the Plastic Behavior of Structures** of the **Pressure Vessel Research Committee** of the **Welding Research Council.**

CONTENTS

1.0 Introduction

In the year 1975, a Task Group on "Characterization of Plastic Behavior of Structures" was organized under the auspices of the Pressure Vessel Research Committee of the Welding Research Council. The group was organized because of the need for unified and standardized methods for limit analysis or plastic collapse determinations. A variety of different methods were being used to determine limit loads and there were questions as to the acceptability of all these different methods.

During 1975–1976, three preliminary studies (*1.1–1.3*) were carried out by the Task Group in three different areas related to three geometries, (plates, shells, and piping and branch connections), where work had been done by other subcommittees of PVRC. After these three summary reports had been prepared, this author was employed in 1976–1978, to review these reports and to prepare the following comprehensive interpretative report on determining limit loads and plastic collapse loads in general.

Four objectives were agreed upon:

1. To review definitions of limit loads and plastic collapse loads as used in both theoretical and experimental analyses, as well as in the ASME code; and to recommend a unified definition of plastic (collapse) load. These definitions were to be treated on two levels: rigid-plastic definitions, and elastic-plastic definitions.
2. To check the definitions and compare theory and experiment in a variety of conditions and on a variety of configurations. The configurations were: beams (as illustrations of basic behavior), pressure vessel heads, unperforated and perforated plates, straight piping, curved piping, nozzles in vessels, and branch connections in pipes. The loadings to be considered were: pressure only, concentrated loads only, moments only and combinations of loadings. The analysis was to be restricted to single monotonic loadings; overloads not cyclic loadings.
3. To assess strain limits, ductility requirements, and shakedown. The range of applicability of limit analysis versus fracture mechanics was to be discussed.
4. To recommend critical tests and research needed to fill in gaps in theory and experiment.

This report covers the above four objectives of the investigation. In this introduction, some basic definitions are first given before considering the theory and application of limit analysis and elastic plastic analysis to different geometries.

The importance of definitions and the setting forth of these definitions very early in the report are key to a report of this type. On considering the definition of the limit moment, care should be taken to clearly differentiate between a limit load or moment and a plastic instability load or moment. For example the following words from P. S. Symonds (*1.4*) bring out the subtleties involved in *limit analysis* concepts and the associated plastic *collapse loads*.

> "It is essential to distinguish the practical phenomenon of plastic collapse, as it occurs in real structures or bodies, from the special meaning of collapse which will be used for mathematical analysis. We will use for mathematical purposes, the concept of plastic collapse of an idealized structure, namely, the condition in which *deflections can increase without limit while the load is held constant*. This rarely (if ever) happens in real bodies or structures, and, hence the calculation applies strictly, not to a real structure, but to a hypothetical one, in which neither work hardening nor significant shape changes occur. Nevertheless, a load computed on the basis of this definition, termed the *limit load*, may give a good approximation to the physical plastic collapse load."

This also emphasizes that *limit load* is a mathematic quantity while *plastic collapse loads* are experienced by actual structures.

Compare the above definition with that given by the ASME Boiler and Pressure Vessel Code for plastic instability and one can see that differences in these definitions rest on shades of meaning.

> "The plastic instability load is taken as the one at which deformation increases without bound, or the relation of force and deformation has a horizontal tangent."

1.1 The First Yield Load

Before the limit load P_0 is defined, the first yield load P_y is defined. Sometimes P_y is confused for P_0.

The first yield load P_y is defined here as the load for which the material of the pressure vessel first yields at the most highly stressed point. This load can be determined from an elastic analysis. Because only one point of the material (of zero or infinitesimal volume) is at yield, the surrounding elastic material restrains the vessel from plastic deformation as a whole.

The first yield load does not necessarily correspond to the proportional limit on a load deflection curve. It may be higher or lower due to material nonlinearity, and large deflection effects.

A pressure vessel of ductile material often can withstand pressures or loads above the first yield point depending upon its configuration. This has led to consideration of methods that will allow determination of loads that better represent the plastic capability of the vessel (or structure). The limit load has been defined for this purpose.

1.2 The Limit Load

The classical definition of a limit load (P_0) according to limit analysis is an idealized one, a mathematical one. A rigid-perfectly-plastic material with a sharply defined yield point is assumed for convenience in the analysis.

In this idealization, elastic portions of material are represented by rigid material. A point in such a material is either rigid with stresses below yield or it is plastic with stresses at yield. At loads above the first yield load, $P > P_y$, but less than the limit load, $P < P_0$, a region of material may have stresses at yield, but this region is still restrained by the remaining rigid portions of material in the vessel. When the load is increased to the limit value P_0, the plastic region has grown to an extent such that the rigid region has either disappeared or has become insufficient to restrain the plastic region from motion. The load for which overall plastic deformation of the vessel occurs is called the limit load. According to limit analysis theory, it is impossible to have loads greater than the limit load for a perfectly plastic material.

Thus, an appropriate definition for the "theoretical limit load" is the maximum load solution to an analytical model of the structure which embodies the following conditions: (*1*) the strain-displacement relations are those of small displacement theory; (*2*) the material response is rigid plastic or elastic-perfectly-plastic, with an admissable yield function; and (*3*) the internal stresses and applied forces are related by the usual equations of equilibrium which ignore changes in geometry due to deformations.

Unfortunately, this load has also been called "a yield-point load" (*1.5*), evidently because the structure as a *whole* yields at this load. But the limit load is *not* the first yield load defined above.

Lower bound solutions and upper bound solutions (*1.4–1.8*) have been developed to bracket the limit load, because they are easier to obtain than exact solutions. A small deflection analysis is used and equilibrium is found about the underformed geometry.

An elastic perfectly-plastic small-deflection analysis can be used to determine the same value of limit load as determined from the rigid perfectly-plastic analysis. The elastic perfectly-plastic analysis will give a value of deflection at which the limit load occurs. In the rigid perfectly-plastic analysis the magnitude of the deflection is undefined, although the strain distribution and the deformation shape are defined.

Both the first-yield load and the limit load are proportional to S_y, the yield strength of the material. This is the only material property used in the analysis. It is assumed that the material is sufficiently ductile so that a plastic analysis applies and so that sensitivity to small notches (scratches) can be ignored. However, geometrical discontinuities are included in the analysis.

1.3 The Plastic Collapse Load

The name, plastic collapse load (P_c), has been applied by Symonds (*1.4*) and others to the actual structure or vessel consisting of an actual strain hardening material. It includes the effects of geometry change due to large deformations. This load is determined experimentally or it can be calculated using an elastic-plastic large-deformation computer analysis. The adjective *collapse* is unfortunate, because the vessel does not necessarily collapse at this load. The terminology of plastic deformation load or just plastic load would be more meaningful, but the terminology of plastic collapse load is prevelant in the literature. The limit load for a perfectly-plastic material is truly a collapse pressure, but not always for actual materials and structures. It is proposed here that the terminology *plastic load* or *plastic pressure* (P_p) be adopted and therefore, these terms will be used in the remainder of this report for actual vessels unless collapse truly occurs.

At this plastic load, significant plastic deformation occurs for the structure or vessel as a whole. It has the same cause as the limit load, i.e., the plastic region in the vessel has grown to a sufficient extent that the surrounding elastic regions no longer prevent overall plastic deformation from occurring. The limit load then can be approximation to the plastic load for the actual vessel, when it is largely plastic at small deflections.

The plastic load depends not only on the yield strength S_y of the material but on its strain hardening modulus as well.

1.4 The Ultimate Load

The limit load or the plastic load are not equal to the ultimate load P_u. The ultimate load depends upon the ultimate strength S_u of the material. The burst pressure is an example of an ultimate pressure for a cylindrical vessel or pipe. Because the ultimate strength of a ductile metal is greater than its yield strength, the ultimate pressure is greater than the limit pressure or the plastic pressure. (It is evident why the adjective collapse should be dropped from the plastic pressure, because the term collapse can be confused with ultimate failure.)

1.5 The Plastic Instability Load

Plastic instability loads P_{pi} can be of two types: (*1*) of the material instability type, and then they correspond to ultimate loads, and (*2*) of the structural instability type. Plastic material instability corresponds for example, to necking of a tensile specimen at the ultimate load.

The plastic instability loads considered in this report are of the structural instability type and depend upon the yield strength S_y of the material, and are accompanied by significant changes in shape of the structure or vessel. The shape change may be axisymmetric in a shell of revolution, and wrinkling (nonaxisymmetric deformation) may or may not occur. The plastic instability load is a particular sub-type of the plastic load that can occur, for example, under external pressure where geometrical weakening occurs (see Chapter 4.0). Calculation of this load requires the capability of a large-deflection elastic-plastic analysis.

The plastic instability load is important because its value is often less than the limit load. At the plastic instability load the load-deflection curve is characterized by a zero slope (horizontal tangent).

1.6 The Shakedown Load

All of the above load definitions are for monotonic increasing loads which is the subject of this report. The shakedown load P_s refers to cyclic loading and although cyclic loading is outside the scope of this report, it is considered briefly because it is important to know the relative margin of safety on shakedown when designing to the limit load or to the plastic load. (The relation is discussed in Chapter 15.)

If upon loading the structure beyond yield into the plastic range to a load value $P > P_y$, and upon unloading a residual stress distribution is produced in the structure such that further cycles of load to value P produce only elastic changes in stress the structure is said to shakedown. The highest value of P for which shakedown occurs is called the shakedown load P_s. Failure to shakedown, i.e., $P > P_s$, leads to either progressive plastic flow called ratcheting, or to low cycle fatigue failure.

Shakedown analyses usually consist of using elasticity theory to determine only lower bounds to P_s, because of the ease of calculation. For further discussion of shakedown, see Chapter 15 and References (*1.6–1.8*).

The main focus of this report is on the determination of limit loads P_0 and plastic loads P_p for various configurations. For the plastic loads many methods have been used to determine estimates. These various methods are evaluated and compared. Comparisons are also made to the other loads P_y, P_u, P_{pi}, and P_s when and where these loads are considered of critical importance.

First, however, basic plastic behavior in simple tension and then plastic bending of beams are considered as an introduction to the subject of limit analysis and elastic-plastic analysis.

1.7 References in Section 1.0

(*1.1*) Gerdeen, J. C., "Review and Interpretive Report on Limit Analysis, Elastic-Plastic Analysis and Experiments on Shells," submitted to Task Group on Characterization of Plastic Behavior of Structures, PVRC, WRC, April 30, 1976.

(*1.2*) O'Donnell, W. J., "Interpretive Report on Limit Analysis of Flat Circular Plates," submitted to Task Group on Characterization of Plastic Behavior of Structures, PVRC, WRC, May, 1976; revised, May, 1977.

(*1.3*) Rodabaugh, E. C., "Interpretive Report on Limit Analysis and Plastic Behavior of Piping Products," submitted to Task Group on Characterization of Plastic Behavior of Structures, PVRC, WRC, Oct., 1976.

(*1.4*) Symonds, P. S., "Limit Analysis," Chapt. 49, *Handbook of Engineering Mechanics*, W. Flügge (Ed.), McGraw-Hill, New York, 1962.

(*1.5*) Hodge, P. G., Jr., *Limit-Analysis of Rotationally Symmetric Plates and Shells*, Prentice-Hall, Englewood Cliffs, NJ, 1963.

(*1.6*) Hodge, P. G., Jr., *Plastic Analysis of Structures*, McGraw-Hill, New York, 1959.

(*1.7*) Massonnet, C. E. and Save, M. A., *Plastic Analysis and Design*, Vol. 1, "Beans and Frames," Blaisdell, XXXXXXX, 1965.

(*1.8*) Save, M. A. and Massonnet, C. E., *Plastic Analysis and Design of Plates, Shells and Disks*, North-Holland, Amsterdam, 1972.

2.0 Plastic Behavior under Uniaxial Tension

Most readers will be familiar with plastic behavior in tension, but some may not. A lack of clear understanding of the basic material behavior, of course, will lead to confusion when the added complexities of geometry, distributed loadings, and large deflections influence the problem. So this section of the report is included for completeness.

An elastic perfectly-plastic material (with no strain hardening) is assumed first, as shown in Fig. 2.1. This behavior closely represents that of mild steel if the strains are not too large. For stresses $\sigma < S_y$, the yield stress, the stress is proportional to the strain, and the

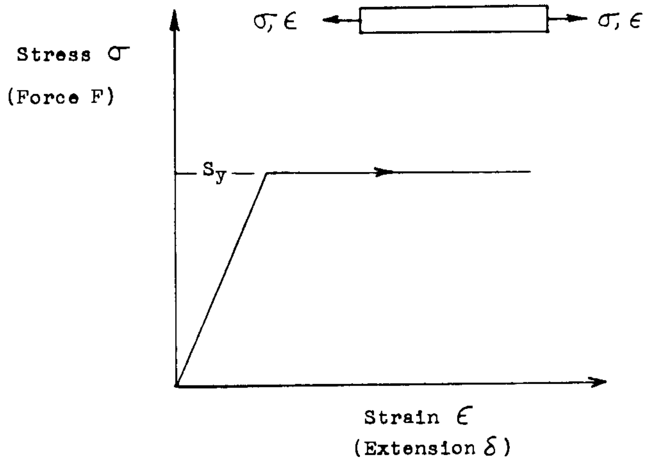

Fig. 2.1—Simple tension behavior for an elastic perfectly-plastic material

strain is limited to $\epsilon = \sigma/E$ where E is the modulus of elasticity. However, when $\sigma = S_y$, the strain ϵ increases without an increase in σ. For this material, the yield stress can also be called a limit stress. No stresses larger than S_y are possible.

If a cylindrical thin-walled pipe were made of this material, and the loading were pure tension F, then Fig. 2.2 would also represent the behavior of F vs. δ, where δ is the axial extension of the pipe—A limit load F_0 can be calculated:

$$F_0 = S_y A = S_y \pi D T, \tag{2.1}$$

where $\sigma = S_y$, and the stress is uniform across the cross section. For $F < F_0$, the extension $\delta = FL/(AE)$ would be limited. However, when $F = F_0$ the extension δ increases without an increase in F, and thus the limit load represents a condition under which gross plastic deformation occurs.

In limit analysis theory, a rigid perfectly-plastic material is usually assumed as an idealization. It is characterized by a sharply defined yield stress, but the elastic strain is neglected as shown in Fig. 2.3. The solution for the limit load, Eq. 2.1 is the same as for the elastic perfectly-plastic material. The solution does not depend on the modulus of elasticity but only on the yield stress S_y. For a rigid plastic material, $\epsilon = 0$ for $\sigma < S_y$ and $\epsilon > 0$ for $\sigma = S_y$.

If the material is strain hardening, the concept of the

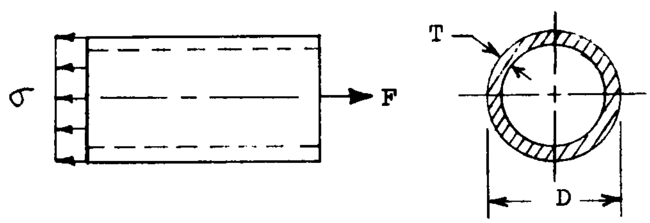

Fig. 2.2—Cylindrical pipe under tension load F

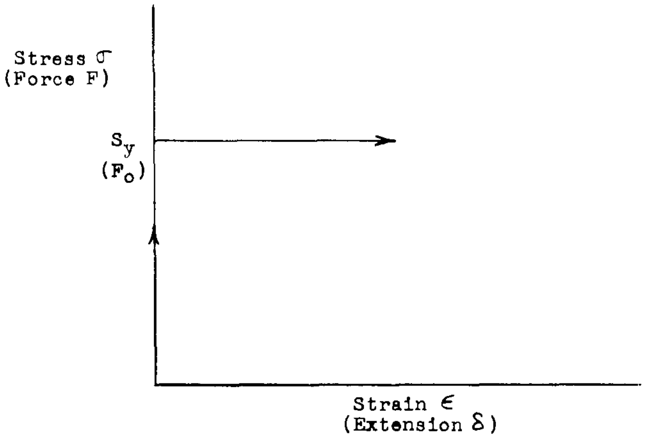

Fig. 2.3—Simple tension behavior for a rigid perfectly-plastic material

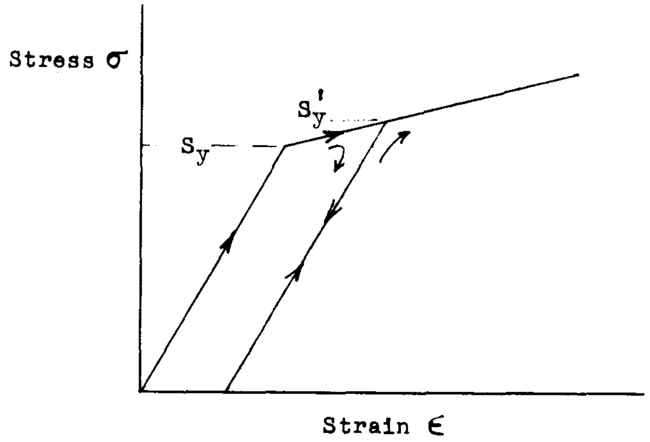

Fig. 2.5—Strain hardening upon loading, unloading and reloading

limit load is still meaningful, however the strain depends on the material hardening phenomenon. Fig. 2.4 shows a strain hardening material. Stresses higher than S_y are possible in this material. The strain hardening is illustrated in Fig. 2.5 where it is shown that if the material is loaded beyond S_y, unloaded and then subsequently reloaded, it has a new yield point S_y.

Assume that a strain-hardening material, Fig. 2.4, has been represented by the perfectly-plastic material. Will the concept of a limit load F_0 still be meaningful? Yes, it will. In Fig. 2.4, assume F_1 below the yield point as shown. A change in F_1 by ΔF causes a change in extension by $\Delta\delta$. However, if $F = F_0$ at S_y, a change in F by ΔF causes a much larger change in extension $\Delta\delta'$ where $\Delta\delta' \gg \Delta\delta$ as shown. Thus, the value F_0 still represents a load at which appreciable plastic deformation will occur with small increases in load.

If, of course, the material is very strain hardening the concept of the limit load F_0 is less meaningful.

The material behavior under compression (with $\sigma < 0$) is assumed to be the same as under tension in this report. Large deflections and geometrical effects will

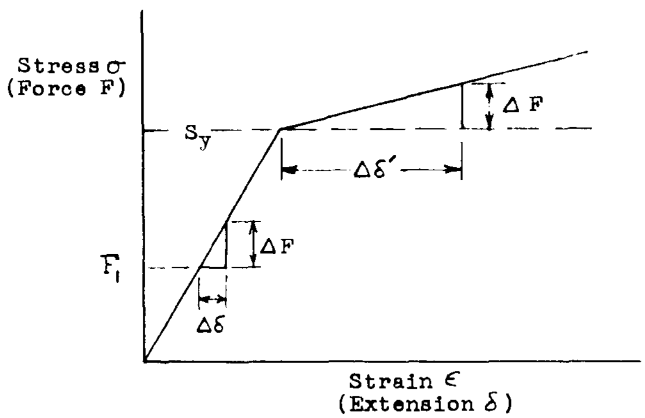

Fig. 2.4—Simple tension behavior for an elastic strain-hardening material

alter the structural behavior, as will be shown later.

Uniaxial tension occurs with a uniform stress across the section. The effects of stress gradients are shown in the next section where bending is considered.

3.0 Plastic Behavior of Beams

The definition of limit loads in structures depends upon an understanding of the concept of yield hinges and the progressive growth of plastic zones of material within the structure. Before considering this complicated phenomenon in pressure vessels and piping, it is illustrated in the simpler geometry of a beam. Small deflections are assumed. The effect of large deflections are considered in Section 4.0.

Beams under transverse loads show the development of "yield hinges" and a difference between the first yield load and the plastic limit load. First, pure bending under constant moment loading is considered. Then a simply supported beam under a concentrated load is presented as an example with one yield hinge, and then a clamped beam under a distributed load is used to show the results when three yield hinges develop in a statistically indeterminant problem. Elastic-plastic behavior is assumed in these analyses. Comparisons are then made with rigid plastic behavior, the basis for the classic "limit load" analysis. Finally, the effect of shear deformation is considered.

3.1 Constant Moment Loading

An elastic perfectly-plastic material is assumed (Fig. 2.1). The loading is a pure moment loading as shown in Fig. 3.1. The moment and the stress are constant along the length of the beam. The yield moment M_y is reached when the stress first reaches yield ($\sigma = S_y$) at the extreme fibers on the top and bottom of the beam. However, all the material between the top and the bottom is still elastic with $\sigma < S_y$, so that moments larger than M_y can be applied. As larger moments are applied, the plastic zone progressively moves in until the whole section is plastic; the top half in compression and the

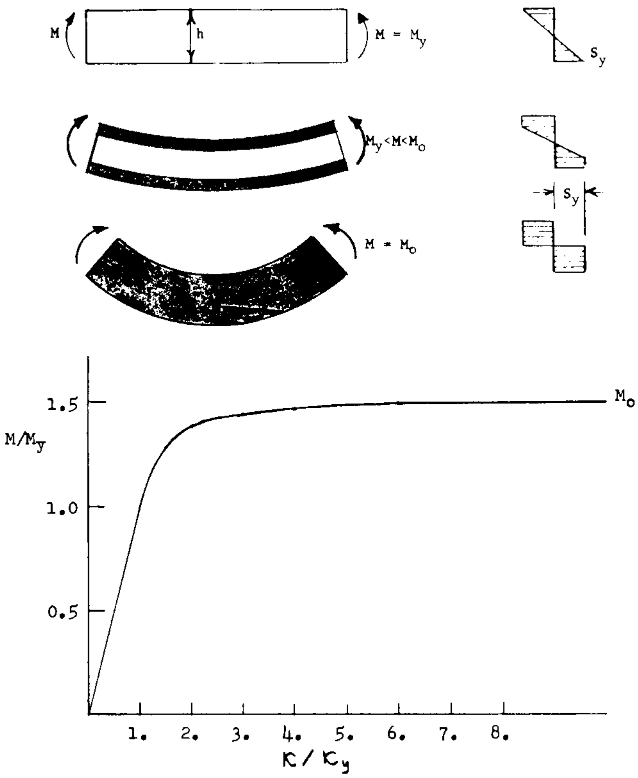

Fig. 3.1—Moment-curvature behavior for a rectangular beam of elastic perfectly-plastic behavior

bottom in tension as shown. The fully plastic moment M_{fp} is the limit moment M_0. It is also called the collapse moment M_c or just the plastic moment M_p in some literature.

From Fig. 3.1, it is evident that the first yield moment M_y is not the limit moment. Plastic yielding at some points in the material does not constitute a limit condition, because the plastic material is constrained by the remaining elastic material. Even though the material itself is perfectly plastic (Fig. 2.1), the beam shows a structural response that looks much like strain hardening (Fig. 2.4) with $M > M_y$ possible, but actually without strain hardening in the material itself. This is due to progressive plastification through the beam because with the stress gradient, all points do not reach yield at the same time.

Moments greater than M_0 are not possible without strain hardening. For the perfectly-plastic case, $M_0 = 1.5\,M_y$ for a rectangular section. For a thin cylindrical pipe,

$$M_0/M_y = f \qquad (3.1)$$

where f is a shape factor defined by

$$f = Z/S, \qquad (3.2)$$

where

$$S = \pi D^3\{1 - (1 - 2T/D)^4\}/32 \qquad (3.3)$$

is the elastic section modulus, and where

$$Z = D^3\{1 - (1 - 2T/D)^3\}/6 \qquad (3.4)$$

is the plastic section modulus. The factor f is 1.40 when $D/T = 10$ and $f \rightarrow 1.27$ when $D/T \rightarrow \infty$. (See Ref. *3.1.*) Thus, the margin between M_0 and M_y depends on D/T.

Note from Fig. 3.1, that as $M \rightarrow M_0$, that the slope of the curve $dM/d\kappa \rightarrow 0$ at $M \rightarrow M_0$. This is the meaning of the limit moment: that relatively large plastic deformations occur when the moment approaches the limit value.

3.2 Simply Supported Beam With Concentrated Load

An elastic-plastic solution is considered. (Later a limit analysis solution will be given.) A perfectly-plastic material, Fig. 2.1 is assumed. The beam is shown in Fig. 3.2 with the plastic zone shown for different values of load. In contrast to the previous problem the moment now varies linearly along the beam, zero at the ends and maximum at the center.

The elastic limit load or equivalently the first yield load P_y is found directly from equilibrium for this statically determinant problem:

$$P_y = 4M_y/L \qquad (3.5)$$

This equation is valid for all loads up to the limit load P_0, (because the problem is statically determinant). Thus

$$P_0 = 4M_0/L \qquad (3.6)$$

For a rectangular beam, $P_0 = 1.5\,P_y$. (For a cylindrical pipe, $P_0 = fP_y$ from Eq. 3.1.) For values of P in between P_y and P_0, the deflection δ under P from Ref. 3.1, p. 42, is given by

$$\delta/\delta_y = (P_y/P)^2\{5 - (3 + P/P_y)\sqrt{3 - 2(P/P_y)}\} \qquad (3.7)$$

for a rectangular beam.

The load-deflection curve represented by (3.7) is shown in Fig. 3.3. Note, that the deflection curve departs

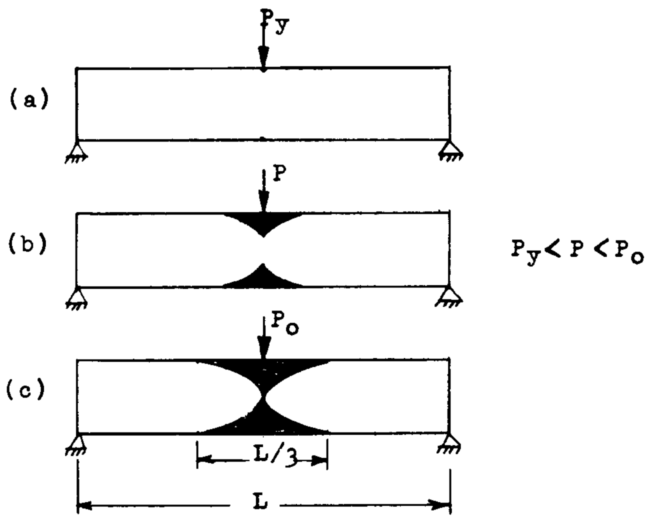

Fig. 3.2—Growth of plastic zone in simply-supported rectangular beam under concentrated load