Annex B
(informative)

Discussion of wave frequency spectra

B.1 The Pierson-Moskowitz spectrum

As noted in A.8.6.3, most parametric spectral formulations developed by oceanographers relate to wind seas, and most of them to fully developed seas. These formulations express the fully developed wave frequency spectrum in terms of the steady state local wind speed only (one-parameter spectrum). Pierson and Moskowitz developed their spectral formulation in 1964 from measured wave data in the North Atlantic in the following form[46]:

$$S_{\mathrm{PM}}(\omega) = \frac{\alpha g^2}{\omega^5} \exp\left[-\beta\left(\frac{g}{\omega U}\right)^4\right] \tag{B.1}$$

where

- α is a numerical constant, $\alpha = 0{,}0081$;
- β is another numerical constant, $\beta = 0{,}74$;
- g is the acceleration of gravity;
- U is the wind speed at 19,4 m above the sea surface.

The factor ω^{-5} controls the high frequency flank of the spectrum, whereas the exponential function controls the low frequency flank. The modal frequency ω_{m} at the peak of the spectrum is defined by $dS(\omega) / d\omega = 0$. This results in

$$\omega_{\mathrm{m}}^4 = \frac{4\beta}{5} \cdot \left(\frac{g}{U}\right)^4$$

or

$$\left(\frac{g}{U}\right)^4 = \frac{5}{4} \cdot \frac{\omega_{\mathrm{m}}{}^4}{\beta} \tag{B.2}$$

Substitution of $(g / U)^4$ from Equation (B.2) into Equation (B.1) results in the Pierson-Moskowitz spectrum as it is usually cited in the literature:

$$S_{\mathrm{PM}}(\omega) = \frac{\alpha g^2}{\omega^5} \exp\left[-\frac{5}{4}\left(\frac{\omega_m}{\omega}\right)^4\right] \tag{B.3}$$

Equation (B.1) may be generalized by releasing the constraints associated with the numerical constants and the dependency on the wind speed by writing it as

$$S_{\mathrm{PM}}(\omega) = \frac{A}{\omega^5} \exp\left(-\frac{B}{\omega^4}\right) \tag{B.4}$$

This transforms the one-parameter Pierson-Moskowitz spectrum (as a function of wind speed) into the two-parameter Pierson-Moskowitz spectrum (as a function of the parameters A and B). The moments of the spectrum (Equation (B.5)) are related to the statistical parameters of the water surface elevation as given by the Equations (B.6). Using these relationships the parameters A and B can be expressed in the significant wave height and a representative frequency or period of the sea state.

The moments of the spectrum and their relationships with A and B in Equation (B.4) are

$$\begin{aligned} m_n &= \int_0^\infty \omega^n S(\omega) d\omega \\ m_0 &= \frac{A}{4B} \\ m_1 &= \frac{\Gamma(3/4)}{4} \cdot \frac{A}{B^{3/4}} \\ m_2 &= \frac{\sqrt{\pi}}{4} \cdot \frac{A}{\sqrt{B}} \\ m_4 &= \infty \end{aligned} \tag{B.5}$$

where $\Gamma(3/4)$ is the gamma function $\Gamma(x)$ for x = 3/4, and $\Gamma(3/4)$ = 1,225 4.

The statistical parameters of the water surface elevation of a random sea and their relationships with the moments of the spectrum are

$$\begin{aligned} H_s &= 4\sqrt{m_0} = 2\sqrt{\frac{A}{B}} \\ \omega_1 &= \frac{2\pi}{T_1} = \frac{m_1}{m_0} = B^{1/4}\Gamma(3/4) \\ \omega_z &= \frac{2\pi}{T_2} = \frac{2\pi}{T_z} = \sqrt{\frac{m_2}{m_0}} = (\pi B)^{1/4} \\ \omega_m &= \frac{2\pi}{T_p} = \left(\frac{4}{5}B\right)^{1/4} \\ \varepsilon &= \sqrt{1 - \frac{m_2^2}{m_0 m_4}} \end{aligned} \tag{B.6}$$

where

T_1 is a mean period of the water surface elevation, defined by the zeroth- and first order spectral moments;

T_2 and T_z are the average zero-crossing period of the water surface elevation, defined by the zeroth- and second order spectral moments, ($T_2 = T_z$);

T_p is the modal or peak spectral period;

ε is the spectral width parameter with $0 \leqslant \varepsilon \leqslant 1{,}0$.

NOTE The modal frequency ω_m is denoted by the subscript m. However, the modal or peak period T_p is denoted by the subscript p rather than m to avoid an erroneous interpretation as the mean period.

The fourth spectral moment m_4 of the Pierson-Moskowitz spectrum [see Equation (B.5)] is infinitely large when the spectrum is integrated from $\omega = 0$ to infinity. This means that $\varepsilon = 1{,}0$ and that the spectrum is broad-banded. In numerical calculations the Pierson-Moskowitz spectrum should always be truncated at a sufficiently high frequency, resulting in a finite m_4 and a (relatively high) value of the spectral width parameter $\varepsilon < 1{,}0$.

Using the Equations (B.6), the parameters A and B can be expressed through H_s and one of the three period options, T_p or $T_z = T_2$ or T_1, all three of which can be found in the literature. All equations can be used with any internally consistent system of units. In SI units, the dimensions of A and B are $m^2(rad/s)^4$ and $(rad/s)^4$, respectively.

Choosing H_s and T_p, the parameters A and B become

$$\begin{aligned} B &= \frac{5}{4}\omega_m^4 = \frac{5}{4}\cdot\left(\frac{2\pi}{T_p}\right)^4 = \frac{20\pi^4}{T_p^4} \\ A &= \frac{BH_s^2}{4} = 5\pi^4\cdot\frac{H_s^2}{T_p^4} \end{aligned} \tag{B.7}$$

The spectral formulation in Equation (B.4) then becomes

$$S_{PM}(\omega) = 5\pi^4\cdot\frac{H_s^2}{T_p^4}\cdot\frac{1}{\omega^5}\cdot\exp\left(-\frac{20\pi^4}{T_p^4}\cdot\frac{1}{\omega^4}\right) \tag{B.8}$$

Choosing H_s and T_z, the parameters A and B become

$$\begin{aligned} B &= \frac{1}{\pi}\omega_z^4 = \frac{1}{\pi}\cdot\left(\frac{2\pi}{T_z}\right)^4 = \frac{16\pi^3}{T_z^4} \\ A &= \frac{BH_s^2}{4} = 4\pi^3\cdot\frac{H_s^2}{T_z^4} \end{aligned} \tag{B.9}$$

and the spectral formulation in Equation (B.4) becomes

$$S_{PM}(\omega) = 4\pi^3\cdot\frac{H_s^2}{T_z^4}\cdot\frac{1}{\omega^5}\cdot\exp\left(-\frac{16\pi^3}{T_z^4}\cdot\frac{1}{\omega^4}\right) \tag{B.10}$$

Finally, choosing H_s and T_1 the parameters A and B become

$$\begin{aligned} B &= \left(\frac{1}{\Gamma(3/4)}\right)^4\cdot\omega_1^4 = \left(\frac{1}{\Gamma(3/4)}\right)^4\cdot\left(\frac{2\pi}{T_1}\right)^4 = 7{,}096\cdot\frac{\pi^4}{T_1^4} \\ A &= \frac{BH_s^2}{4} = 1{,}774\,\pi^4\cdot\frac{H_s^2}{T_1^4} \end{aligned} \tag{B.11}$$

and the spectral formulation in Equation (B.4) becomes

$$S_{PM}(\omega) = 1{,}774\,\pi^4\cdot\frac{H_s^2}{T_1^4}\cdot\frac{1}{\omega^5}\cdot\exp\left(-\frac{7{,}096\,\pi^4}{T_1^4}\cdot\frac{1}{\omega^4}\right) \tag{B.12}$$

Considering the differences between the Equations (B.8), (B.10) and (B.12), care should be taken to combine the correct formulation of the spectrum with the period chosen to represent the sea state. The choice usually depends on the type of available data and user preference. Equating the expressions for the parameter B from the Equations (B.7), (B.9) and (B.11) the relationships between the peak (the modal) period T_p, the average zero-crossing period $T_z = T_2$ and the mean period T_1 for a Pierson-Moskowitz spectrum are found to be

$$\begin{aligned} T_1 &= 1{,}086\ T_z &&= 0{,}772\ T_p \\ 0{,}920\ T_1 &= T_z &&= 0{,}710\ T_p \\ 1{,}296\ T_1 &= 1{,}408\ T_z &&= T_p \end{aligned} \tag{B.13}$$

B.2 The JONSWAP spectrum

The JONSWAP wave frequency spectrum resulted from extensive measurements taken off the coast of the German island of Sylt[47]. The JONSWAP spectrum is formulated as a modification of the Pierson-Moskowitz spectrum for a developing sea state in a fetch limited situation:

$$S_{JS}(\omega) = F_n\ S_{PM}(\omega) \left\langle \gamma^{\exp\left\{-\frac{1}{2}\left[(\omega-\omega_m)/(\sigma\,\omega_m)\right]^2\right\}} \right\rangle \tag{B.14}$$

where

γ is a non-dimensional peak shape parameter;

σ is a numerical parameter

$\sigma = \sigma_a$ for $\omega \leqslant \omega_m$

$\sigma = \sigma_b$ for $\omega > \omega_m$

F_n is a normalizing factor used to ensure that both spectral forms have the same H_s.

For $\gamma = 1$ the JONSWAP spectrum reduces to the Pierson-Moskowitz spectrum. The factor in the large brackets in Equation (B.14) is a peak enhancement factor, which is a function of the three parameters γ, σ_a and σ_b. These parameters were not constant in the North Sea data obtained in the project but showed appreciable scatter. Average values from the JONSWAP data were

$$\begin{aligned} \gamma &= 3{,}3, \\ \sigma_a &= 0{,}07, \\ \sigma_b &= 0{,}09. \end{aligned} \tag{B.15}$$

The peak shape parameter γ varied between about 1 and 6 and was approximately normally distributed with a mean of 3,3 and a standard deviation of 0,79. The JONSWAP spectral form also appears to be capable of representing the observations rather well in different geographical areas, provided that the parameters γ, σ_a and σ_b are chosen in accordance with the local data, see Reference [36]. The values in other areas are likely to be very different from the JONSWAP data.

For $\gamma > 1,0$ the peak enhancement factor is always larger than 1,0 for all ω; therefore $S_{JS}(\omega) \geqslant S_{PM}(\omega)$ for all ω. Without the normalizing factor F_n, the JONSWAP spectrum would hence have a larger energy content (a larger H_s) than the corresponding Pierson-Moskowitz spectrum; its modal frequency is also larger: $\omega_{m,JS} \geqslant \omega_{m,PM}$. To ensure that H_s is the same for both spectra, the normalizing factor should be

$$F_n = \frac{\int_0^\infty S_{PM}(\omega)d\omega}{\int_0^\infty S_{PM}(\omega)\left\langle \gamma^{\exp\left\{-\frac{1}{2}\left[(\omega-\omega_m)/(\sigma\omega_m)\right]^2\right\}}\right\rangle d\omega} \tag{B.16}$$

As the JONSWAP spectrum cannot be integrated analytically, the normalizing factor can only be calculated numerically. Based on curve fitting through the results of a number of numerical exercises for different peak shape parameter values, but always using σ_a = 0,07 and σ_b = 0,09, the expressions for the normalizing factor shown in Equation (B.17) were developed:

$$\begin{aligned} F_n(1) &= \left[0,78 + 0,22\gamma\right]^{-1} && \text{for } 1 \leqslant \gamma \leqslant 6 \\ F_n(2) &= \left[5\left(0,065\gamma^{0,803} + 0,135\right)\right]^{-1} && \text{for } 1 \leqslant \gamma \leqslant 10 \end{aligned} \tag{B.17}$$

$F_n(1)$ was obtained by Ewing[48] and $F_n(2)$ by Yamaguchi[49].

By way of example, for different values of γ the expression $F_n(2)$ results in

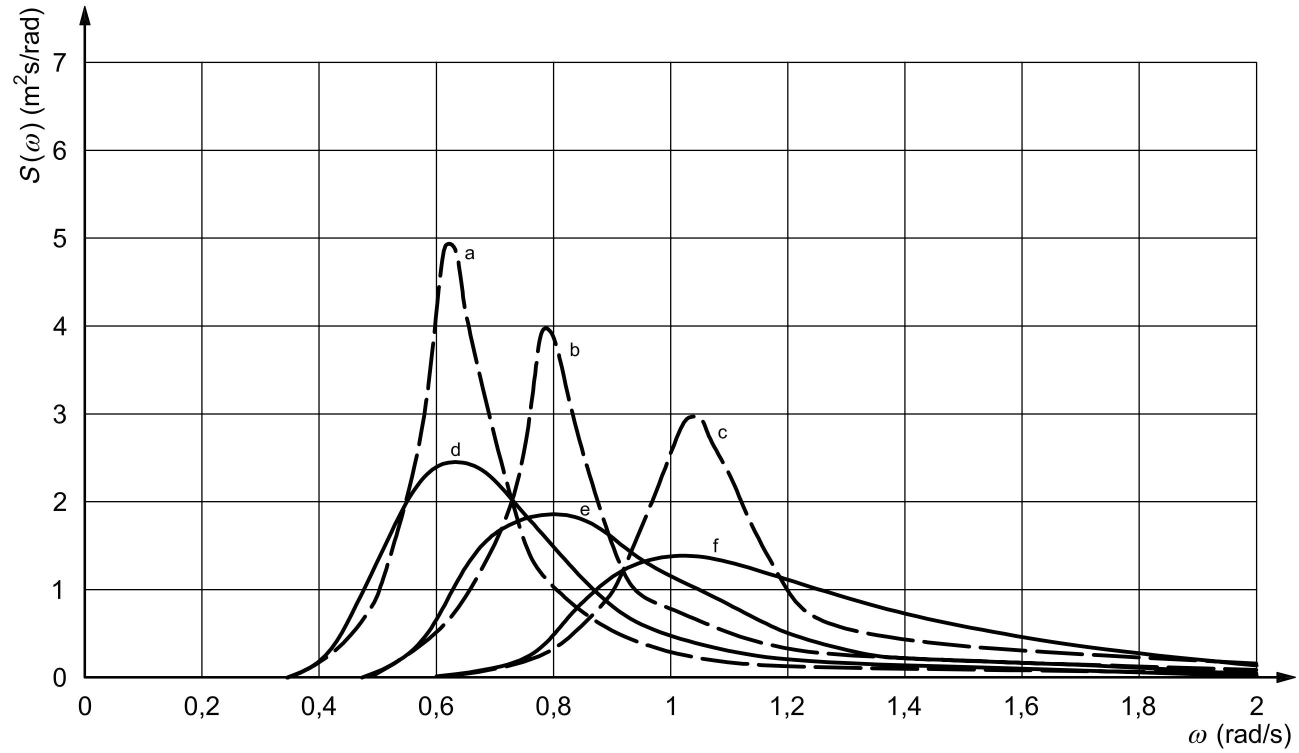
$$\begin{aligned} \gamma &= 1 && F_n(2) = 1,00 \\ &= 2 && F_n(2) = 0,81 \\ &= 3 && F_n(2) = 0,68 \\ &= 5 && F_n(2) = 0,54 \\ &= 10 && F_n(2) = 0,36 \end{aligned} \tag{B.18}$$

Numerical integration of the JONSWAP spectrum for the average values of γ = 3,3; σ_a = 0,07 and σ_b = 0,09 results in the following ratios between T_p, $T_z = T_2$ and T_1:

$$\begin{array}{llll} & T_1 & = 1,073\ T_z & = 0,834\ T_p \\ 0,933 & T_1 & = \quad T_z & = 0,777\ T_p \\ 1,199 & T_1 & = 1,287\ T_z & = \quad T_p \end{array} \tag{B.19}$$

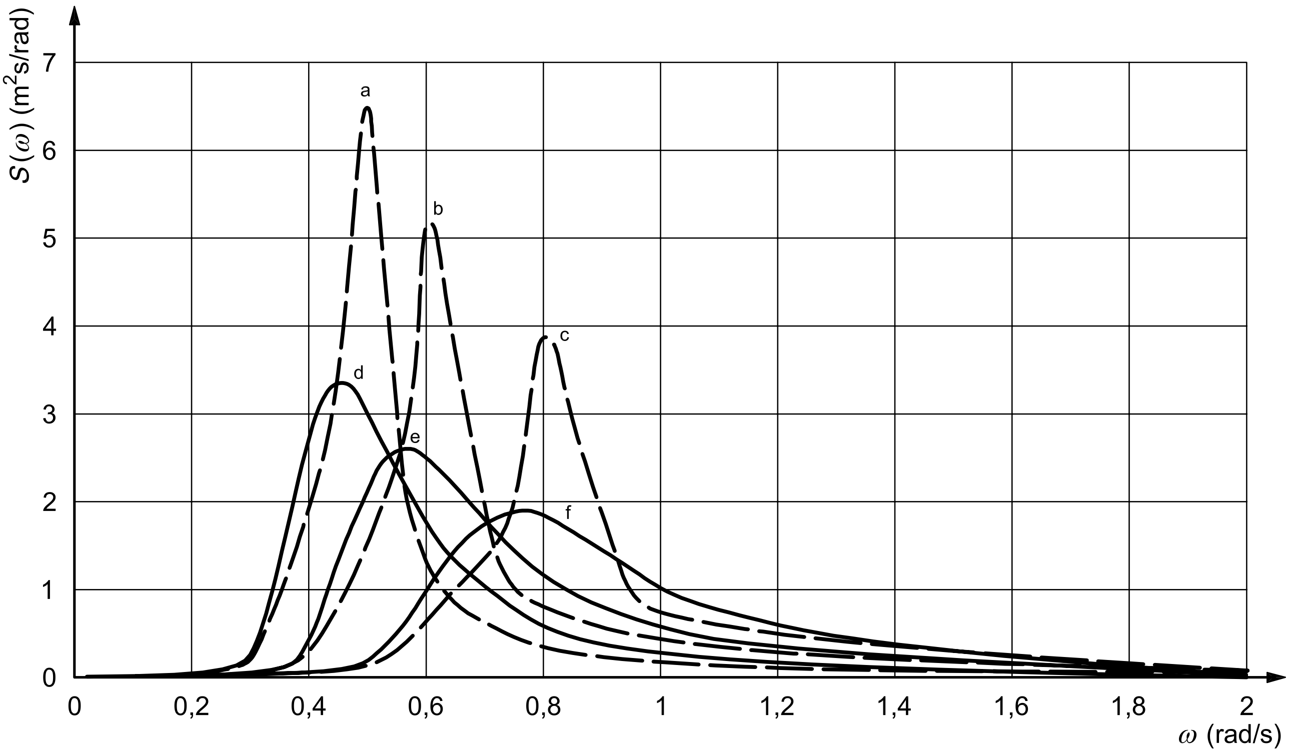
B.3 Comparison of Pierson-Moskowitz and JONSWAP spectra

For illustration, Figures B.1 and B.2 show a comparison of the Pierson-Moskowitz and JONSWAP spectral formulations for three different sea states each. The JONSWAP spectra are based on the average project data of γ = 3,3; σ_a =0,07; σ_b = 0,09; F_n = 0,66. The significant wave height is H_s = 4,0 m for all sea states. In Figure B.1 the spectral peak periods of both formulations are the same (T_p = 6 s, 8 s and 10 s respectively). In Figure B.2 the mean zero-crossing periods of both formulations are the same (T_z = 6 s, 8 s and 10 s respectively); the relationship between mean zero-crossing period and peak period for the JONSWAP spectrum is T_p = 1,287 T_z, in accordance with Equation B.19. Note the different distribution of wave energy over frequency for corresponding Pierson-Moskowitz and JONSWAP spectra, as well as the shift in position of the spectra between the figures.



a JONSWAP, T_p = 10 s
b JONSWAP, T_p = 8 s
c JONSWAP, T_p = 6 s
d Pierson-Moskowitz, T_p = 10 s
e Pierson-Moskowitz, T_p = 8 s
f Pierson-Moskowitz, T_p = 6 s

Figure B.1 — Pierson-Moskowitz and JONSWAP spectra — H_s = 4,0 m **— Equal peak periods:** T_p = 6 s, 8 s, 10 s



a JONSWAP, T_z = 10 s
b JONSWAP, T_z = 8 s
c JONSWAP, T_z = 6 s
d Pierson-Moskowitz, T_z = 10 s
e Pierson-Moskowitz, T_z = 8 s
f Pierson-Moskowitz, T_z = 6 s

Figure B.2 — Pierson-Moskowitz and JONSWAP spectra — H_s = 4,0 m **— Equal mean zero-crossing periods:** T_z = 6 s, 8 s, 10 s

B.4 Ochi-Hubble spectra

Ochi-Hubble spectra[37] are a general spectral formulation to describe seas which consist of a combination of two different sea states, each of which is in turn described by a further generalization of the Pierson-Moskowitz spectrum including three instead of two parameters. Ochi-Hubble spectra thus have six parameters in total. The discussion below is based on Reference [36].

The Pierson-Moskowitz spectrum can be normalized by dividing it by its zeroth moment, which results in

$$S_{\text{PM,n}}(\omega) = \frac{S_{\text{PM}}(\omega)}{m_0(\omega)} = \frac{4}{\omega^5}\left[B\exp\left(-\frac{B}{\omega^4}\right)\right] \tag{B.20}$$

where

$S_{\text{PM,n}}(\omega)$ is the normalized Pierson-Moskowitz spectrum;

$S_{\text{PM}}(\omega)$ is the Pierson-Moskowitz spectrum of Equation (B.4);

$m_0(\omega)$ is the zeroth moment of the Pierson-Moskowitz spectrum of Equation (B.5).

As $S_{PM,n}(\omega)$ has unit area, Equation (B.20) may be considered as if it were a probability density function. The factor within the square brackets has the form of the probability density function of the exponential distribution:

$$f_{exp}(x) = \alpha \exp(-\alpha x) \tag{B.21}$$

$x \geqslant 0$

$\alpha > 0$

with $\alpha = B$ and $x = \omega^{-4}$. The exponential distribution is a special case of the more general gamma distribution with the probability density function:

$$f_{gam}(x) = \frac{1}{\Gamma(\lambda)} x^{\lambda-1} \alpha^{\lambda} \exp(-\alpha x) \tag{B.22}$$

$x \geqslant 0$

$\alpha > 0$

$\lambda > 0$

For $\lambda = 1$, the gamma distribution reduces to the exponential distribution. The normalized Pierson-Moskowitz spectrum $S_{PM,n}(\omega)$ may hence be generalized to become $S_{gen,n}(\omega)$ by substituting the gamma probability density function for the exponential probability density function in Equation (B.20), which results in

$$S_{gen,n}(\omega) = \frac{4}{\omega^5}\left[\frac{1}{\Gamma(\lambda)} \omega^{-4(\lambda-1)} B^{\lambda} \exp\left(-\frac{B}{\omega^4}\right)\right] = \frac{4}{\Gamma(\lambda)} \frac{B^{\lambda}}{\omega^{4\lambda+1}} \exp\left(-\frac{B}{\omega^4}\right) \tag{B.23}$$

The parameter B can be determined by observing that the spectrum has a horizontal tangent at the spectral peak, i.e. $d(S_{gen,n}(\omega)) / d\omega = 0$ for $\omega = \omega_m$. This provides the one and only solution:

$$B = \frac{4\lambda+1}{4} \omega_m^4 \tag{B.24}$$

This is the equivalent of Equation (B.6) for the Pierson-Moskowitz spectrum. Substitution of B from Equation (B.24) into Equation (B.23) gives the generalized spectral formulation as

$$S_{gen,n}(\omega) = \frac{4}{\Gamma(\lambda)}\left(\frac{4\lambda+1}{4}\omega_m^4\right)^{\lambda} \frac{1}{\omega^{4\lambda+1}} \exp\left[-\frac{4\lambda+1}{4}\left(\frac{\omega_m}{\omega}\right)^4\right] \tag{B.25}$$

$S_{gen,n}(\omega)$ is still normalized with unit area. To describe a sea state with a significant wave height H_s, it should be multiplied by $H_s^2 / 16$, see Equation (B.6), finally resulting in

$$S_{gen}(\omega) = \frac{H_s^2}{4\Gamma(\lambda)}\left(\frac{4\lambda+1}{4}\omega_m^4\right)^{\lambda} \frac{1}{\omega^{4\lambda+1}} \exp\left[-\frac{4\lambda+1}{4}\left(\frac{\omega_m}{\omega}\right)^4\right] \tag{B.26}$$

$S_{gen}(\omega)$ is a more general spectral formulation than the Pierson-Moskowitz spectrum, having three instead of two parameters, i.e. H_s, $\omega_m = 2\pi / T_p$ and λ. It is easily verified that for $\lambda = 1$, the spectrum $S_{gen}(\omega)$ reduces to the Pierson-Moskowitz spectrum.

The Ochi-Hubble spectra are obtained by combining two spectra of the form of Equation (B.26), one for the low frequency components (usually a swell) and one for the high frequency components of the wave energy (usually a wind sea), see Figure B.3. The spectral formulation of the Ochi-Hubble spectra is accordingly

$$S_{\mathrm{OH}}(\omega) = S_{\mathrm{gen},1}(\omega) + S_{\mathrm{gen},2}(\omega) = \sum_{j=1,2} \left\{ \frac{H_{\mathrm{s},j}^4}{4\Gamma(\lambda_j)} \left(\frac{4\lambda_j + 1}{4} \omega_{\mathrm{m},j}^4 \right)^{\lambda_j} \frac{1}{\omega^{4\lambda_j + 1}} \exp\left[-\frac{4\lambda_j + 1}{4} \left(\frac{\omega_{\mathrm{m},j}}{\omega} \right)^4 \right] \right\} \quad \text{(B.27)}$$

It should be noted that each of the two general spectra has one peak only [i.e. unimodal, see Equation (B.24)]. However, Ochi-Hubble spectra are combinations of two spectra and can obviously have two peaks (i.e. bimodal). This is not necessarily always the case; while there will clearly be a “hump” in the total spectrum at the location of $\omega_{\mathrm{m},2}$, the sum of the spectral ordinates between $\omega_{\mathrm{m},1}$ and $\omega_{\mathrm{m},2}$ might well be larger than the peak value of the high frequency spectrum at $\omega_{\mathrm{m},2}$ so that the tangent at $\omega_{\mathrm{m},2}$ not need be horizontal.

Another property of the combination of two spectra is

$$m_{\mathrm{n,OH}}(\omega) = \int_0^\infty \omega^n S_{\mathrm{OH}}(\omega)\,\mathrm{d}\omega = \int_0^\infty \omega^n \left[S_{\mathrm{gen},1}(\omega) + S_{\mathrm{gen},2}(\omega) \right] \mathrm{d}\omega = m_{\mathrm{n},1}(\omega) + m_{\mathrm{n},2}(\omega) \quad \text{(B.28)}$$

From which it follows that

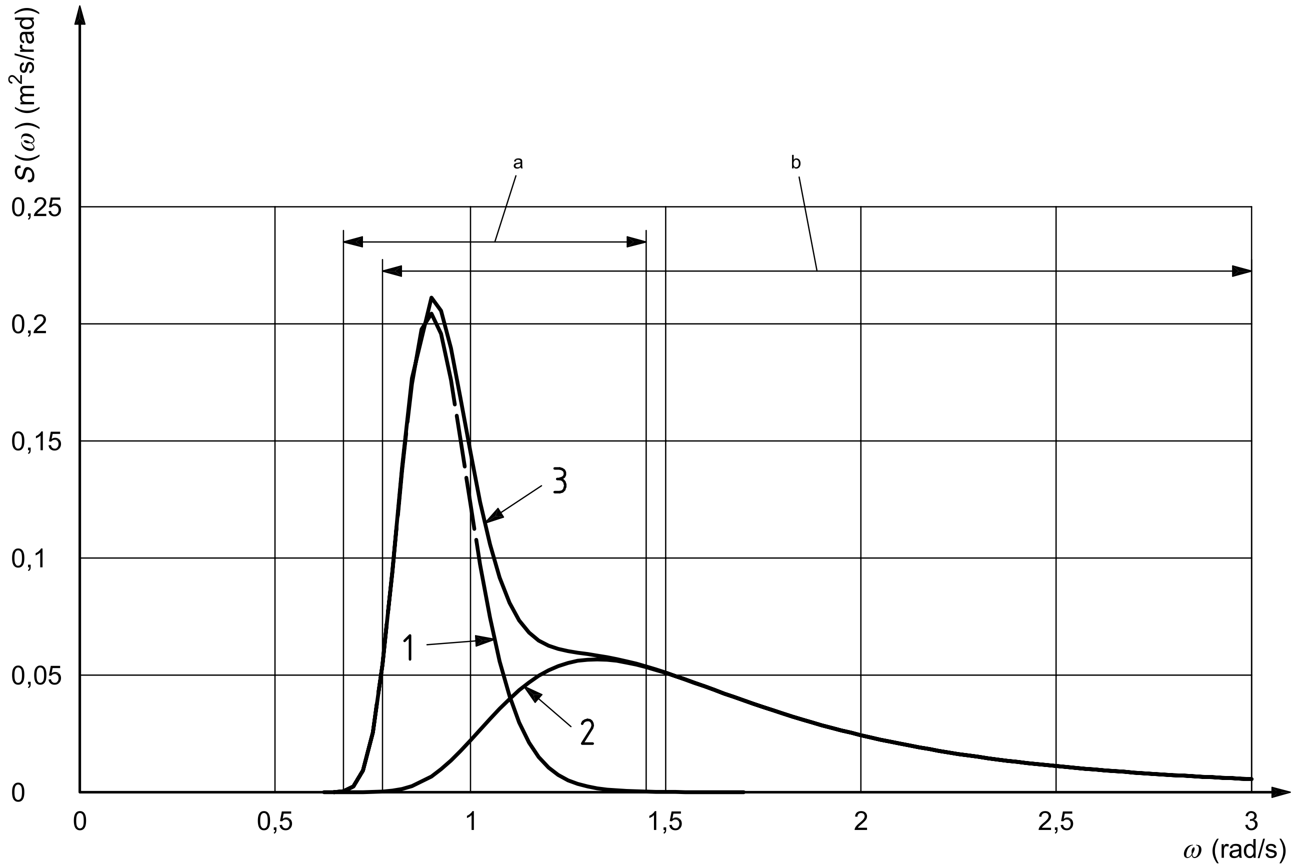
$$H_{\mathrm{s}}^2 = H_{\mathrm{s},1}^2 + H_{\mathrm{s},2}^2 \quad \text{(B.29)}$$

where

H_{s} is the total significant wave height of the combined sea state;

$H_{\mathrm{s},1}$ is the significant wave height of the low frequency part of the sea state;

$H_{\mathrm{s},2}$ is the significant wave height of the high frequency part of the sea state.



Key

ω	frequency
$S(\omega)$	spectrum
1	swell spectrum
2	wind sea spectrum
3	total spectrum

[a] Frequency range of swell spectrum.

[b] Frequency range of wind sea spectrum.

Figure B.3 — Ochi-Hubble spectrum — Swell parameters: $H_{s,1}$ = 0,875 m, $T_{p,1}$ = 7 s, λ_1 = 6 **— Wind sea parameters:** $H_{s,2}$ = 1,0 m, $T_{p,2}$ = 4,75 s, λ_2 = 0,75

Annex C
(informative)

Regional information

C.1 General

This annex presents an overview of various regions of the world for which information has been developed by experts on each region, and is intended to supplement the provisions, information and guidance given in the main body and Annexes A and B of this part of ISO 19901. It also provides some guidance relating to the particular region dealt with in each of its clauses, as well as some indicative values for metocean parameters which can be suitable for conceptual studies. However, site- or project-specific shall be developed for structural design and/or assessment.

C.2 North-west Europe

C.2.1 Description of region

The geographical extent of the region of north-west Europe is bounded by the continental shelf margins of Europe as shown in Figure C.1. The region is diverse, stretching from the sub-arctic waters off Norway and Iceland to the Atlantic seaboard of France and Ireland in the south, and includes

- the waters off Norway, part of which are within the Arctic Circle,
- the Baltic Sea,
- the North Sea,
- the Irish Sea,
- the English Channel,
- the northern half of the Bay of Biscay,
- the waters off the west coasts of Ireland and Scotland, and
- the waters off the Faeroes Islands.

C.2.2 Data sources

Measured data are available from many stations throughout the area. Sources for measured data may be identified through the International Oceanographic Data and Information Exchange[50], which is part of UNESCO[1)]. Links will be found to national oceanographic data centres, which in turn provide links to specialist institutes and other organizations within each country. Data may also be obtained from commercial organizations. In addition to measured data, in recent years a number of joint, industry-sponsored hindcast studies have been performed — see for example, References [51] and [52]. These have resulted in extensive (but usually proprietary) data sets for the companies involved; however, a recently published report[53] provides useful information derived from the NEXT hindcast study[51].

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1) United Nations Educational, Scientific and Cultural Organization.