low-loss cables. In waveguide systems there is little that can be done to eliminate this error except to calibrate the phase shift of the movable arms and rotary joints as a function of position in space if such a calibration is required.

These examples should indicate to the experimenter that accurate measurements require careful attention to the details of the system, and assumed accuracies of say 5° or less shall be viewed with caution unless all factors have been taken into account. One factor that is sometimes overlooked is the signal-to-noise ratio in the system. A high signal-to-noise ratio is necessary to prevent the noise from contributing to errors in the phase measurements.

11. Polarization

11.1 General

Polarization is a property of single-frequency electromagnetic radiation describing the shape and orientation of the locus of the extremity of the field vectors as a function of time [1, 3.1–3.4], [70].

NOTE — Some of the methods of analysis used for single-frequency fields can be extended to partially polarized fields [71], [72]. Random fields or random antennas will not be considered here.

In common practice, when only plane waves or locally plane waves are considered, it is sufficient to specify the polarization of the electric field vector *E*. In a plane wave with a known direction of propagation the magnetic field vector *H* is simply related to the *E* field. It can be deduced by a 90° rotation about the propagation vector, followed by multiplication by the intrinsic admittance Y_0 of the medium,

$$Y_0 = \sqrt{\frac{\varepsilon}{\mu}}$$

where $\in$ is the permittivity and μ is the permeability of the medium. In vacuum $Y_0 \approx 1/377 = 2.66 \times 10^{-3}\ \Omega^{-1}$.

The far field radiated by an antenna is generally observed in a small region where it can be considered as a plane wave propagating away from the antenna in the radial direction. The electric field is in a plane perpendicular to that direction. The locus of its extremity, is, in general, an ellipse that may degenerate into a segment of a straight line or into a circle. Correspondingly, the polarization is called elliptical, linear, or circular.

The sense of rotation of the extremity of *E* describing a circle or an ellipse in the plane of polarization (perpendicular to the direction of propagation) is called the sense of polarization, or handedness. This sense is called right handed (left handed) if the direction of rotation is clockwise (counterclockwise) for an observer looking in the direction of propagation (see Fig 40). Alternative conventions are discouraged as they could be a source of confusion. If, instead of considering the field vector at a point as a function of time, one considers the vector at a given instant of time, as a function of distance along the direction of propagation, one might be lead to incorrect designations.

Elliptical polarization is characterized by the axial ratio of the polarization ellipse, the sense of rotation, and the spatial orientation of the ellipse with respect to a reference direction in the plane containing the ellipse. The angle between the reference direction and the major axis of the ellipse is called the *tilt angle*.

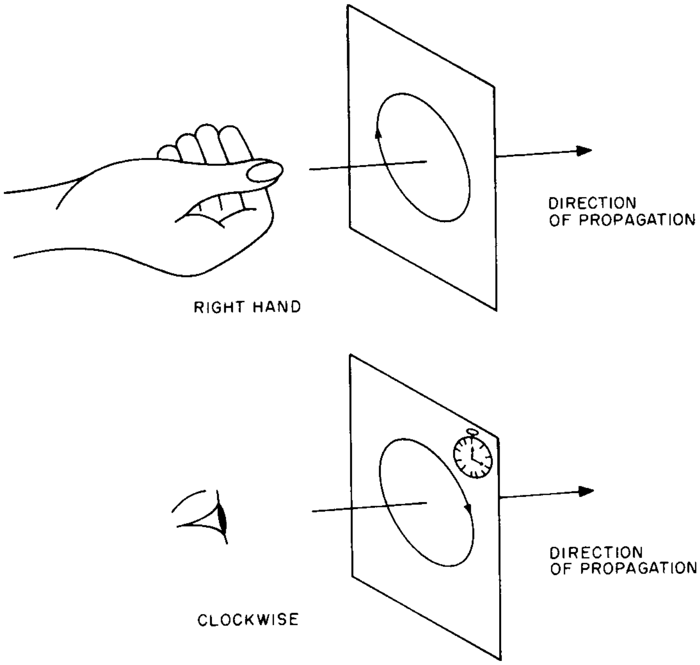

Figure 40—Illustration of the Sense of Rotation

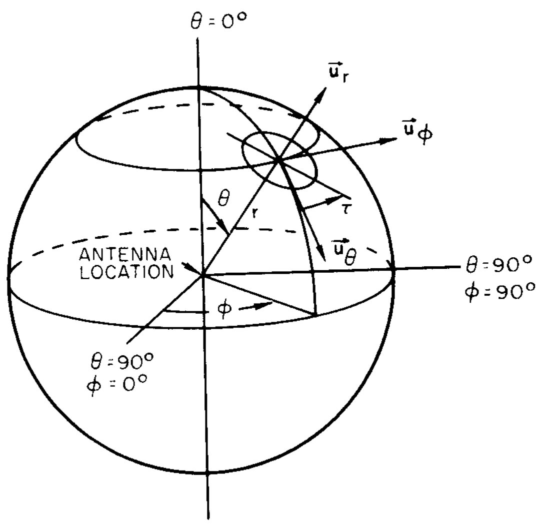

Figure 41—Polarization Ellipse in Relation to Antenna Coordinate System

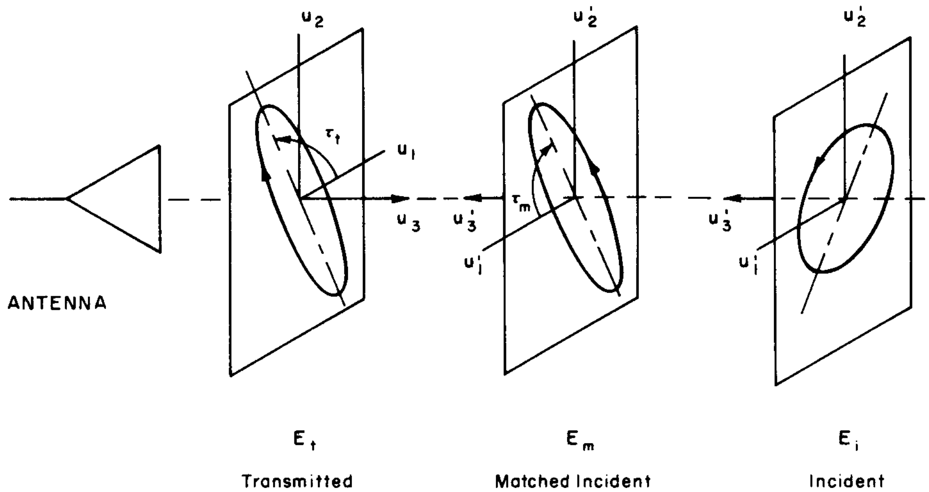

Figure 42—Relation Between Polarization Properties of an Antenna when Transmitting and Receiving
E_t — Far-Field Electric Vector of Antenna; E_m — Electric vector of Incident Wave which Is Polarization Matched to Antenna; E_i — Electric Vector of Arbitrarily Polarized Incident Wave

For a plane wave, the tilt angle is measured clockwise from the reference direction when the plane of polarization is viewed in the direction of propagation.

The polarization of an antenna in a given direction is defined as the polarization of the electric field vector E_t in the far field, radiated by the antenna in that direction. Since an antenna usually has a spherical coordinate system associated with it (see 3.1), its polarization in a direction (θ, ϕ) can be illustrated with respect to the coordinate system, as shown in Fig 41. Although the reference direction for establishing the orientation of the polarization ellipse is arbitrary, it is common practice to take the $\boldsymbol{u}_\theta$ axis as the reference direction [2]. For most antenna-pattern-measurement situations it is convenient to establish a local coordinate system in a plane perpendicular to a line drawn between the test antenna and the source antenna (see 3.1 and 3.2) with one axis horizontal and the other vertical with respect to the range surface [73, pp 475–477]. The horizontal axis is usually chosen as the reference direction. This convention shall be employed throughout this standard.

When an antenna receives a plane wave coming from a given direction, the response (open-circuit voltage, short-circuit current, or available power) is maximum, for a given intensity of the incident plane wave, when the polarization ellipse of the incident electric field E_m has the same axial ratio, the same sense of polarization, and the same spatial orientation of the major axis as that of the antenna for that direction (see Fig 42). Because the sense of polarization is relative to the direction of propagation, and because this direction is different for E_t and E_m, the senses appear to be different when looked at from a single point of view. Also, in order to be consistent with the definition of antenna polarization, the local coordinate system associated with each of the waves is oriented so that one of the coordinates is in the direction of propagation. As a result the tilt angles for the two polarization ellipses, which are measured according to a right-hand rule, are different. As shown in Fig 42 if τ_t is the tilt angle for the ellipse described by E_t, then the one described by E_m will be

$$\tau_m = 180° - \tau_t$$

It should be noted that τ_m may also be expressed as $-\tau_t$ plus any integral multiple of 180° since all these angles correspond to the same orientation of the major axis of the polarization ellipse. The polarization of the incident wave, which yields the maximum response at the antenna terminals, as described above, is called the *receiving polarization* of the antenna.

If the incident plane wave has a polarization that is different from the receiving polarization of the antenna, then a polarization loss occurs due to this mismatch. The *polarization efficiency p* is used to account for polarization mismatch [70, pp 544–549], [74].

NOTE — This factor is also called polarization mismatch factor [75], and polarization receiving factor (ANSI/IEEE Std 100-1977 , Dictionary of Electrical and Electronics Terms).

It is defined as the ratio of the power actually received by the antenna divided by the power that would be received if a wave from the same direction, with the same intensity and polarization matched, were incident on the antenna.

The Poincaré sphere [70, pp 540–544] is a useful graphical aid for the visualization of polarization effects since the polarization efficiency is uniquely determined by the separation of two points on the sphere which describe the polarizations of the incident wave and the receiving antenna, respectively.

The construction of the Poincaré sphere is consequence of the fact that any wave can be resolved into two orthogonal component (they may be two orthogonal linear, elliptical, or circularly polarized components). The total power in the wave can then be represented as the sum of the powers contained in the orthogonal components:

$$Y_0 E^2_{\mathrm{t}} = Y_0 E^2_A + Y_0 E^2_B$$

where E_{t} represents the effective value of the magnitude of the electric field vector of the given wave, and E_A and E_B are those of the two orthogonal components. By dividing the equation by the total power $Y_0 E^2_{\mathrm{t}}$ one obtains

$$\overline{E}^2_A + \overline{E}^2_B = 1$$

where ^2_A and ^2_B represent the fractions of power in their respective polarizations. This latter equation leads to the construction shown in Fig 43. It is evident in Fig 43 that the position of *W* along the arc *AWB* indicates the division of power in the wave *W* between the two orthogonal polarizations *A* and *B*.

For example, if *W* represents the incident plane wave and *A* the antenna's receiving polarization, then the angular separation 2ζ between *A* and *W* determines the division of power density in the wave between the receiving polarization and the orthogonal polarization. The polarization efficiency *p* is given by

$$p = \overline{E}^2_A = \cos^2 \zeta$$

Note that $p = 1$ when *A* and *W* are coincident and 0 when *A* and *W* are diametrically opposite. For the former case the wave is said to be copolarized with respect to the antenna's receiving polarization and for the latter case they are cross-polarized. Fig 44 depicts the relative spatial orientation of the polarization ellipses of two cross-polarized fields. Note that for elliptically polarized fields a rotation of 90° about the direction of propagation alone does not produce a cross-polarized field. The rotation shall be accompanied by a change of the sense of polarization.

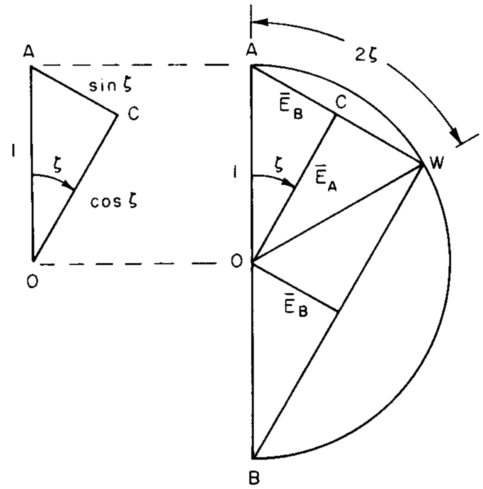

Figure 43—Illustration of the Division of Power Between Two Orthogonal Elliptical Polarizations A and B

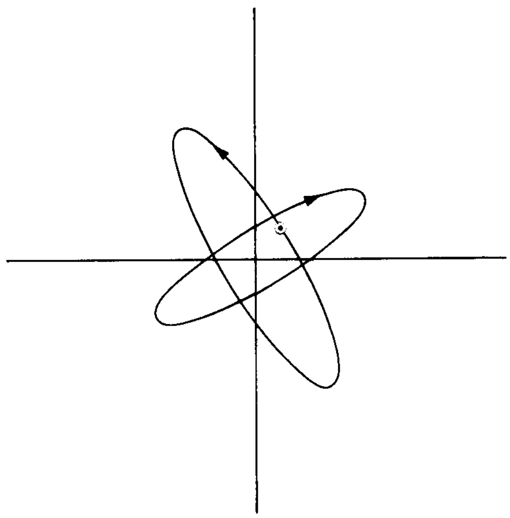

Figure 44—Cross-Polarized Field Vectors

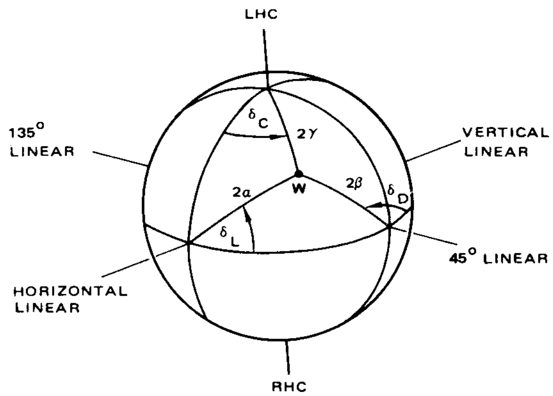

Figure 45—Poincaré Sphere Representation of the Polarization of a Plane Wave W

The Poincaré sphere is generated by rotating the semicircle *AWB* about the *A-B* axis with the angular measure around the equator of the resulting sphere being the relative phase between the two orthogonal polarizations *A* and *B* (see Fig 45). In Fig 45 the angle 2ζ of Fig 43, which was defined for arbitrary elliptical components *A* and *B*, is assigned the symbols 2α, 2β, and 2γ as the polarizations *A* and *B* become

$$2\alpha \begin{cases} \bar{E}_H & \text{(horizontal linear)} \\ \bar{E}_V & \text{(vertical linear)} \end{cases}$$

$$2\beta \begin{cases} \bar{E}_{45} & \text{(45° linear)} \\ \bar{E}_{135} & \text{(135° linear)} \end{cases}$$

$$2\gamma \begin{cases} \bar{E}_L & \text{(left-hand circular (LHC))} \\ \bar{E}_R & \text{right-hand circular (RHC)} \end{cases}$$

There is a one-to-one correspondence between all possible polarizations and points on the Poincaré sphere (see Fig 46).

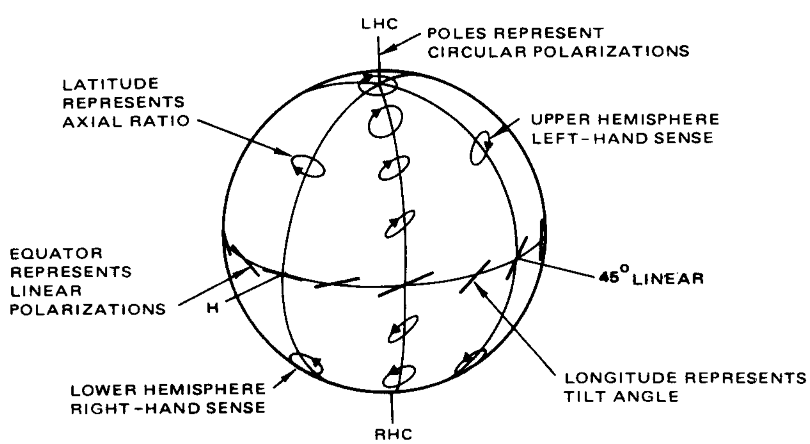

Figure 46—Representation of Polarization on the Poincaré Sphere

The *complex polarization ratios* are given by

$$_{L} = \rho_L e^{j\delta_L}$$

$$_{D} = \rho_D e^{j\delta_D}$$

$$_{C} = \rho_C e^{j\delta_C}$$

where

$$\rho_L = \tan \alpha = {}_{V}/_{H}$$

$$\rho_D = \tan \beta = {}_{135}/_{45}$$

$$\rho_C = \tan \gamma = {}_{R}/_{L}$$

and where δ_L, δ_D, and δ_C are the relative phases between the corresponding orthogonal components. The relative phase δ_C of the circularly polarized components is defined by the angle of the instantaneous electric vector of the right-hand circular component with respect to the horizontal direction at the instant that the electric vector of the left-hand circular component is in the horizontal direction,

as shown in Fig 47. Hence the two circularly polarized components are in phase when the electric field vectors in these two waves are in the same horizontal direction at the same time. The circular polarization ratio ρ_C is of particular interest since the axial ratio r of the polarization ellipse may be expressed as

$$r = \frac{\rho_C + 1}{\rho_C - 1}$$

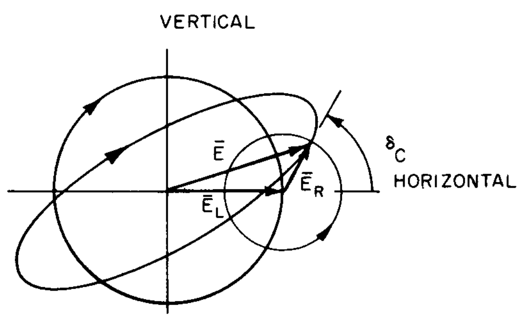

Figure 47—Definition of Phase Reference for Orthogonal Circular Components

Note that the sign of the denominator gives the sense of polarization with r positive for the right-hand sense and negative for the left-hand sense. Also, the tilt angle τ is given by

$$\tau = \delta_C/2$$

If the points corresponding to the receiving polarization of the antenna A_r and the polarization of the incident wave W are located on the Poincaré sphere, then the polarization efficiency can be determined by use of the expression

$$p = \cos^2\zeta$$

where 2ζ is the angular separation between A_r and W as shown in Fig 48. Other forms for p can be obtained with the use of the polarization ratios. For example, it can be shown that

$$p = \frac{1 + \rho_W^2\rho_r^2 + 2\rho_W\rho_r \cos \Delta}{(1 + \rho_W^2)\ (1 + \rho_r^2)}$$

where ρ_W and ρ_r can be any of the three polarization ratios ρ_L, ρ_D, or ρ_C, with Δ being the corresponding difference in phase angles $[(\delta_L)_W - (\delta_L)_r]$, $[(\delta_D)_W - (\delta_D)_r]$, or $[(\delta_C)_W - (\delta_C)_r]$. When the circular polarization ratio ρ_C is chosen, the angle Δ is twice the spatial angle between the tilt angles of the polarization ellipses of the two polarizations. This equation for p can also be written as

$$p = \frac{|1 + \hat{\rho}_W\hat{\rho}_r^*|^2}{(1 + \rho_W^2)\ (1 + \rho_r^2)}$$

where $_W$ and $_r$ are the complex polarization ratios. This form is interesting because it is similar to the equation for mismatch loss in lossless transmission lines with the source and load reflection coefficients being analogous to ρ_W and ρ_r, respectively (see 12.5).

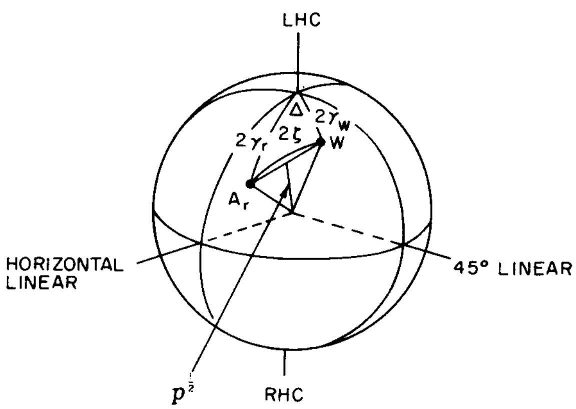

Figure 48—Polarizations of Incident Wave W and Receiving Antenna A_r, Plotted on the Poincaré Sphere

The polarization efficiency can also be written in terms of the axial ratios for the two polarizations by the use of the relationship between the circular polarization ratio ρ_C and the axial ratio:

$$p = \frac{(1 + r_W^2)(1 + r_r^2) + 4r_W r_r + (1 - r_W^2)(1 - r_r^2) \cos \Delta}{2(1 + r_W^2)(1 + r_r^2)}$$

In this formula the axial ratios are taken as positive for right-hand polarization and negative for left-hand polarization. The angle Δ is the difference between the relative phases $(\delta_C)_{W,r}$ for the two polarizations W and A_r.

The Poincaré sphere and the polarization box [1, pp 3.1–3g.7], which is a graphical representation of the Stokes parameters, provide a convenient means of converting from one set of polarization parameters to another. The polarization box is shown in Fig 49 which also indicates its relationship to the Poincaré sphere. It can be shown that the sides and the side diagonals are given by

Sides	**Side Diagonals**
$Y_L = \cos 2\alpha$	$X_L = \sin 2\alpha$
$Y_D = \cos 2\beta$	$X_D = \sin 2\beta$
$Y_C = \cos 2\gamma$	$X_C = \sin 2\gamma$

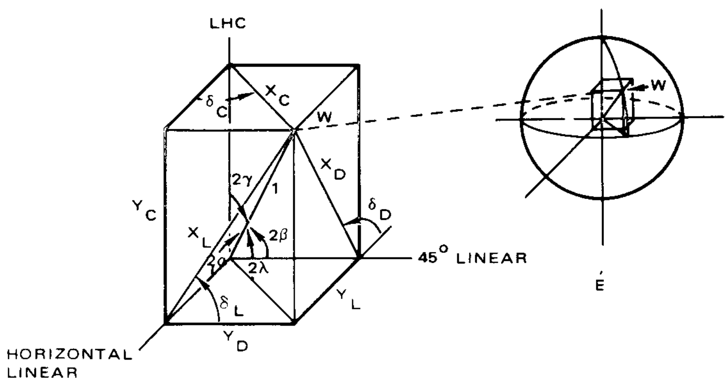

Figure 49—Polarization Box and Its Relation to the Poincaré Sphere

As an example of the use of the polarization box in converting between polarization parameters, let ρ_L and δ_L be known from measurements, and let it be required to find ρ_C, δ_C, γ, and the tilt angle τ. The solution is as follows. From the polarization box it can be seen that

$$Y_C = X_L \sin \delta_L$$

so that

$$\cos 2\gamma = \sin 2\alpha \sin \delta_L$$

From the definition of ρ_L one can compute α:

$$\alpha = \tan^{-1} \rho_L$$

Thus γ can be determined, from which ρ_C can be calculated by the use of

$$\rho_C = \tan \gamma$$

and

$$\delta_C = \cos^{-1}\left[\frac{Y_L}{X_C}\right] = \cos^{-1}\left[\frac{\cos 2\alpha}{\sin 2\gamma}\right]$$

The ambiguity in defining δ_C from its cosine alone is removed by inspection of the polarization box. Finally, once ρ_C and δ_C are known, the axial ratio r and the tilt angle τ can be obtained by use of

$$r = \frac{\rho_C + 1}{\rho_C - 1}, \quad \tau = \delta_C / 2$$

For some applications it is advantageous to express the polarization of a field in the form of a unitary vector. A wave W, for example, can be expressed in terms of two orthogonal elliptical polarizations as

$$W = \begin{bmatrix} \bar{E}_A \\ \bar{E}_B\ e^{j\delta_E} \end{bmatrix} = \begin{bmatrix} \cos \zeta \\ \sin \zeta\ e^{j\delta_E} \end{bmatrix}$$

where $_A$, $_B$, and ζ are defined in Fig 43, and δ_E is the relative phase between $_A$ and $_B$. A and B can become (H, V), [45], [135], or (L, R), in which case ζ and δ_E become (α, δ_L), (β, δ_D), or (γ, δ_C), respectively.

To illustrate the use of polarization vectors, let the incident wave W and the antenna receiving polarization A_r be expressed in terms of the circular polarization components, that is,

$$W = \begin{bmatrix} \cos \gamma_W \\ \sin \gamma_W e^{j(\delta_C)_W} \end{bmatrix}$$

$$A_r = \begin{bmatrix} \cos \gamma_r \\ \sin \gamma_r e^{j(\delta_C)_r} \end{bmatrix}$$

The normalized voltage response $\overline{V}$ of the antenna to the wave is proportional to the inner product of A_r and W which can be expressed as

$$\overline{V} = (A_r, W) = A_r^{\dagger} W$$

where the superscript † denotes the complex conjugate of the transpose. (Note that is a phasor.) The matrix operation yields

$$\overline{V} = \cos \gamma_r \cos \gamma_W + \sin \gamma_r \sin \gamma_W e^{j\Delta}$$

where

$$\Delta = (\delta_C)_W - (\delta_C)_r = 2\,(\tau_W - \tau_r)$$

The polarization efficiency is then given by

$$p = * = ||^2$$

The determination of the polarization efficiency by the vector method is a formalization of the process of resolving the power density of the incoming wave and the effective aperture of the antenna each into two components corresponding to two orthogonal polarizations such that each component of the wave is polarization matched to the corresponding components of the antenna's effective aperture. It should be noted that partial responses due to the pairs of polarization-matched components do not, in general, add to give a maximum value of $\overline{V}$. The condition for a polarization match is that $\gamma_r = \gamma_W$ and $\Delta = 0$, in which case

$$p = ||^2 = (\cos^2 \gamma + \sin^2 \gamma)^2 = 1$$

11.2 Polarization Measurements

11.2.1 General

The radiation pattern of an antenna designed for a specific polarization is usually described in terms of the field components for that polarization. It is only a partial description, since a cross-polarized component may be present. A complete description of the radiation pattern requires the measurement of polarization as a function of direction. In particular, away from the direction of the peak value of the main beam, the polarization may be quite different from the design value and, even over the main beam, its variation may be of importance.

The various techniques used to measure the polarization of antennas may be broadly classified into three categories:

1) those that yield partial information about the antenna's polarization properties
2) those that yield complete polarization information, but require comparison with a polarization standard
3) those that yield complete polarization information, but require no standard or *a priori* knowledge of the polarizations of the antennas used in the measurement

The methods of category (2) are termed transfer or comparison methods, and those of category (3) are referred to as absolute methods. The method one selects depends upon the type of test antenna, the required accuracy, the amount of polarization data required, the time available for the measurements, and the permissible cost.

The following methods may be employed to measure polarization [1, pp 10.1–10.38]:

1) polarization-pattern method
2) rotating-source method
3) multiple-amplitude-component method
4) phase-amplitude method

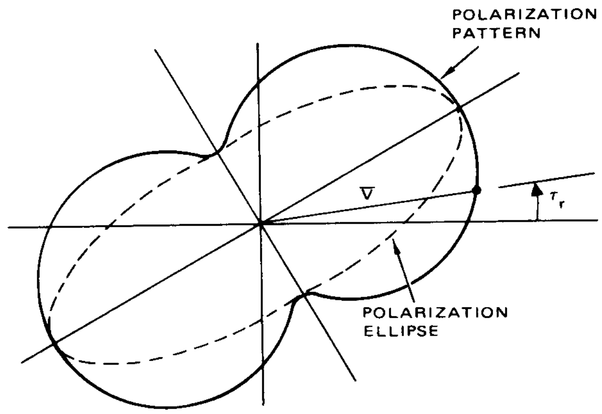

Figure 50—Polarization Pattern of a Wave

To completely characterize the polarization of an antenna, it is necessary to determine the polarization ellipse (axial ratio and tilt angle) and the sense of rotation of the electric field vector of the wave radiated by the antenna. The polarization state of the wave can be represented as a unique point on the Poincaré sphere (see 11.1). Some methods of polarization measurement yield insufficient information to completely characterize the state of the wave, hence a unique point on the Poincaré sphere is not determined. For example, when the axial ratio and tilt angle are measured, but the sense of rotation is not determined, there will be an ambiguity between conjugate points in the upper and lower hemispheres of the Poincaré sphere. This information may be adequate when the sense of polarization is otherwise known or when the polarization is nearly linear so that the two conjugate points lie close to the equator.