For $(A + B) < X \leq (A + B + C)$

$$\delta_X = \frac{P(A+B+C-X)}{4EI_C}\left\{C^2 + \frac{(B+C)^2 - C^2}{K_2} + \frac{(A+B+C)^2 - (B+C)^2}{K_3} - \frac{(A+B+C-X)^2}{3}\right\}$$

Where:

$P = F_R$ = radial force acting at impeller location (I: consideration of radial thrust only), in N

$P = F_R + M_I \times g$ = radial force acting at impeller location (II: consideration of radial thrust and impeller weight), in N

M_I = impeller mass, in kg

$A, B, C, D_A, D_B, D_C, X$ = dimensions per Figure 14.3.6.4.2.2, in mm (in)

I_A, I_B, I_C = area moment of inertia $[I = \frac{\pi}{64}D^4]$, in mm^4 (in^4)

E = Young's Modulus of Elasticity of shaft material, in N/mm^2 (psi)

K_2 = $\frac{I_B}{I_C}$ = shaft area moment ratio, *B* to *C* (dimensionless)

K_3 = $\frac{I_A}{I_C}$ = shaft area moment ratio, *A* to *C* (dimensionless)

NOTES:

1) These calculations do not consider contributions of any shaft sleeve to the stiffness of the shaft, and the additional mass has negligible effect on deflection.

2) Equations do not account for any support the pump shaft might receive from hydrostatic stiffness at the impeller wear rings and seal chamber throat bushing.

3) Equations do not account for internal looseness in the bearings, dynamic radial loads caused by impeller imbalance, or shaft runout.

14.3.6.4.3 Critical speed

14.3.6.4.3.1 Method of calculating dry critical speed for pumps with overhung impellers (neglecting coupling weight)

Shaft deflection at impeller location (X = 0) δ_X:

$$\delta_X = \frac{M_I g}{3E}\left\{\frac{Z^2 A}{I_A} + C_3 X\left(\frac{1}{I_C} - \frac{1}{I_B}\right) + \frac{Z^3}{I_B}\right\} \text{ mm (in)}$$

Where:

M_I = impeller mass, in kg (lb)

g = 9.81 m/s^2 (32.2 ft/s^2), gravitational constant

E = Young's Modulus of Elasticity of shaft material, in N/mm^2 (psi)

A, B, C, D_A, Z = dimensions per illustration above, in mm (in)

I_A, I_B, I_C = polar area moment of inertia (e.g., $I_A = \frac{\pi D_A^4}{64}$)), in mm^4 (in^4)

Shaft stiffness at impeller location K_S:

$$P = M_I g = K_s \delta_X$$

$$K_S = \frac{M_I g}{\delta_X} = \frac{3E}{\left\{\frac{Z^2 A}{I_A} + C^3\left(\frac{1}{I_C} - \frac{1}{I_B}\right) + \frac{Z^3}{I_B}\right\}} \text{ N/mm (lbf/in)}$$

Dry critical speed f_{DRY}:

$$f_{DRY} = \frac{1}{2\pi}\left(\frac{K_S}{M_I}\right)^{0.5} \text{ Hz}$$

Sample calculation – (metric units) (see Figure 14.3.6.4.2.1)

Where:

A = 186.5 mm

B = 216 mm

C = 43 mm

D_A = 65 mm

D_B = 48 mm

D_C = 28 mm

M_I = 4.8 kg

E = 2.069 × 10^5 N/mm^2

Z = $B + C$ = 216 + 43 = 259 mm

$I_A = \frac{\pi D_A^4}{64} = \frac{\pi \times 65^4}{64}$ = 8.762 × 10^5 mm^4. Similarly, I_B = 2.606 × 10^5 mm^4 and I_C = 3.017 × 10^4 mm^4.

Shaft deflection at the impeller ($X = 0$) is

$$\delta_X = \frac{4.8 \times 9.81}{3 \times 2.069 \times 10^5}\left\{\frac{259^2 \times 186.5}{8.762 \times 10^5} + 43^3\left(\frac{1}{3.017 \times 10^4} - \frac{1}{2.606 \times 10^5}\right) + \frac{259^3}{2.606 \times 10^5}\right\}$$

$$\delta_X = 6.319 \times 10^{-3} \text{ mm}$$

Shaft stiffness at the impeller location is

$$K_S = \frac{M_I g}{\delta_X} = \frac{4.8 \times 9.81}{6.319 \times 10^{-3}} = 7.451 \times 10^{-3} \text{ N/mm}$$

Dry critical speed is

$$f_{DRY} = \frac{1}{2\pi}\left(\frac{7.541 \times 10^{3} \times 1000}{4.8}\right)^{0.5} = 198.3 \text{ Hz}$$

Sample calculation – (US customary units) (see Figure 14.3.6.4.2.1)

Where:

A = 7.34 in

B = 8.50 in

C = 1.69 in

D_A = 2.56 in

D_B = 1.89 in

D_C = 1.10 in

M_I = 10.582 lb

E = 3.000×10^{7} psi

Z = $B + C = 8.50 + 1.69 = 10.20$ in

$I_A = \frac{\pi D_A^4}{64} = \frac{\pi \times 2.56^4}{64} = 2.105 \text{ in}^4$ Similarly, $I_B = 6.261 \times 10^{-1}$ in^4 and $I_C = 7.250 \times 10^{-2}$ in^4.

Shaft deflection at the impeller ($X = 0$) is

$$\delta_X = \frac{10.582}{3 \times 3.000 \times 10^{7}}\left\{\frac{10.20^2 \times 7.34}{2.105} + 1.69^3\left(\frac{1}{7.250 \times 10^{-2}} - \frac{1}{6.261 \times 10^{-1}}\right) + \frac{10.20^3}{6.261 \times 10^{-1}}\right\}$$

$\delta_X = 2.487 \times 10^{-4}$ in

Shaft stiffness at the impeller location is

$$K_S = \frac{MI}{\delta_X} = \frac{10.582}{2.487 \times 10^{-4}} = 4.255 \times 10^{4} \text{ lb/in}$$

Dry critical speed is

$$f_{DRY} = \frac{1}{2\pi}\left(\frac{4.255 \times 10^{4} \times 386.22}{10.582}\right)^{0.5} = 198.3 \text{ Hz}$$

14.3.6.4.3.2 Method of calculating dry critical speed for pumps with impeller(s) between bearings (neglecting coupling weight)

Shaft deflection at impeller location ($X = 0$) δ_O

$$\delta_0 = \frac{MI_I g}{6EI_C}\left\{C^3 + \frac{(C+B)^3 - C^3}{K_2} + \frac{(A+B+C)^3 - (B+C)^3}{K_3}\right\} \text{mm (in)}$$

Where:

M_I = impeller mass, in kg (lb)

g = 9.81 m/s² (32.2 ft/s²), gravitational constant

E = Young's Modulus of Elasticity of shaft material, in N/mm² (psi)

A, B, C = dimensions per Figure 14.3.6.4.2.2, in mm (in)

I_A, I_B, I_C = polar area moment of inertia (e.g., (e.g., $I_A = \frac{\pi D_A^4}{64}$)), in mm⁴ (in⁴)

Shaft stiffness at impeller location K_S:

$$P = M_I\,g = K_S\,\delta_X$$

$$K_S = \frac{M_I g}{\delta_X} = \frac{6EI_C}{\left\{\frac{Z^2 A}{I_A} + C^3\left(\frac{1}{I_C} - \frac{1}{I_B}\right) + \frac{Z^3}{I_B}\right\}} \text{N/mm (lbf/in)}$$

$$K_S = \frac{M_I g}{\delta_X} = \frac{6EI_C}{\left\{\frac{C^3 + (C+B)^3 - C^3}{K_2} + \frac{(A+B+C)^3 - (B+C)^3}{K_3}\right\}} \text{N/mm (lbf/in)}$$

Dry critical speed f_{DRY}:

$$f_{DRY} = \frac{1}{2\pi}\left(\frac{K_S}{M_I}\right)^{0.5} \text{Hz}$$

Sample calculation – (metric units) (see Figure 14.3.6.4.2.2)

Where:

A = 240 mm

B = 170 mm

C = 140 mm

D_A = 90 mm

D_B = 80 mm

D_C = 75 mm

M_I = 100 kg

E = 2.10×10^5 N/mm²

Shaft seal location = 320 mm from impeller centerline

$$I_A = \frac{\pi D_A^4}{64} = \frac{\pi \times 90^4}{64} = 3.221 \times 10^6 \text{ mm}^4\text{. Similarly, } I_B = 2.011 \times 10^6 \text{ mm}^4 \text{ and } I_C = 1.553 \times 10^6 \text{ mm}^4.$$

Shaft area moment ratio, from B to C is

$$K_2 = \frac{I_B}{I_C} = \frac{2.011 \times 10^6}{1.553 \times 10^6} = 1.295$$

Shaft area moment ratio, from A to C is

$$K_3 = 2.074$$

Shaft deflection at the impeller centerline ($X = 0$) is

$$\delta_0 = \frac{100 \times 9.81}{6 \times 2.10 \times 10^5 \times 1.553 \times 10^6}\left\{140^3 + \frac{(140+170)^3 - 140^3}{1.295} + \frac{(240+170+140)^3 - (170+140)^3}{2.074}\right\}$$

$$\delta_0 = 4.487 \times 10^{-2} \text{ mm}$$

Shaft stiffness at the impeller centerline ($X = 0$) is

$$K_S = \frac{M_I g}{\delta_0} = \frac{100 \times 9.81}{4.487 \times 10^{-2}} = 2.186 \times 10^4 \text{ N/mm}$$

Dry critical speed is

$$f_{DRY} = \frac{1}{2\pi}\left(\frac{2.186 \times 10^4 \times 1000}{100}\right)^{0.5} = 74.4 \text{ Hz}$$

Sample calculation (US customary units) (see Figure 14.3.6.4.2.2)

Where:

A = 9.449 in

B = 6.693 in

C = 5.512 in

D_A = 3.543 in

D_B = 3.150 in

D_C = 2.953 in

M_I = 220.5 lb

E = 3.05×10^7 psi

Shaft seal location = 12.599 in from impeller centerline

$$I_A = \frac{\pi D_A^4}{64} = \frac{\pi \times 3.543^4}{64} = 7.738 \text{ in}^4\text{. Similarly, } I_B = 4.831 \text{ in}^4 \text{ and } I_C = 3.731 \text{ in}^4.$$

$$K_2 = \frac{I_B}{I_C} = \frac{4.831}{3.731} = 1.295\text{. Similarly, } K_3 = 2.074.$$

Shaft deflection at the impeller centerline ($X = 0$) is

$$\delta_0 = \frac{220.5}{6 \times 3.05 \times 10^7 \times 3.731}\left\{5.512^3 + \frac{(5.512 + 6.693)^3 - 5.512^3}{1.295} + \frac{(9.449 + 6.693 + 5.512)^3 - (6.693 + 5.512)^3}{2.074}\right\}$$

$$\delta_0 = 0.0018 \text{ in}$$

Shaft stiffness at the impeller centerline ($X = 0$) is

$$K_S = \frac{M_I g}{\delta_0} = \frac{220.5}{0.0018} = 1.225 \times 10^5 \text{ lbf/in}$$

Dry critical speed is

$$f_{DRY} = \frac{1}{2\pi}\left(\frac{1.225 \times 10^5 \times 386.22}{220.5}\right)^{0.5} = 73.7 \text{ Hz}$$

As an alternative to the hand calculations, finite element analysis (FEA) can be used to analyze the complete shaft loading. This includes not only the deflection due to bending but torsional deflection as well. Care must be taken to accurately mesh the shaft model to fully capture the effects of the shaft features such as keyways, snap ring grooves, fillets, and threaded portions. These can create regions of stress concentration that can reduce the overall load capacity of the shaft. These not only affect the shaft stress in a steady state condition, but often more importantly, in the fatigue life of the shaft.

14.3.6.5 Design of vertical lineshaft

14.3.6.5.1 Stress limits for vertical turbine lineshafting

The Hydraulic Institute recognizes and adopts the ANSI/AWWA E103-07 *Standard for Horizontal and Vertical Line-Shaft Pumps (VTP)* for pump shafting power transmission design. This is a nationally recognized VTP industry standard. (See current edition for formulae and design details.) The design stress safety margins have been successfully used since the 1960s. The standard states the pump shafting maximum combined shear stress shall not exceed 30% of the material yield tensile strength (YTS) nor be more than 18% of the ultimate tensile strength (UTS).

The formulae for determining combined shaft shear stress are as follows:

Metric units:

$$S_{CS} = (1 \times 10^3) \times \sqrt{\left(\frac{2 \times F_A}{\pi \times D^2}\right)^2 + \left(\frac{4.86 \times 10^7 \times P}{n \times D^3}\right)^2}$$

US customary units:

$$S_{CS} = \sqrt{\left(\frac{2 \times F_A}{\pi \times D^2}\right)^2 + \left(\frac{321{,}000 \times P}{n \times D^3}\right)^2}$$

Where:

S_{CS} = combined shear stress, in kPa (psi)

F_A = axial thrust acting through the shafting, in N (lb)

D = shaft diameter at the root of the threads or minimum diameter at any undercut, in mm (in)

P = power transmitted by the shaft, in kW (hp)

n = rotational speed of pump, in rpm

The lineshafts shall be of a material and size that will transmit the torque from the driver to the impellers and support the maximum thrust load with a proper factor of safety.

The threaded ends of lineshafts should be connected with a shaft coupling that has a factor of safety greater than that of the shaft. Threads at these joints shall be such that they tighten when operating.

On larger-diameter shafts, threaded connections can be difficult to assemble or disassemble. Alternatively, a coupling connection built to accommodate keys to transmit torque and split thrust rings or other means to transmit thrust should be used.

14.3.6.5.2 Stress limits for vertical turbine threaded lineshaft couplings

ANSI/AWWA E103-07 *Standard for Horizontal and Vertical Line-Shaft Pumps (VTP)* states a threaded shaft coupling maximum combined shear stress shall not exceed 20% of the material YTS nor be more than 12% of the UTS.

The formula for determining threaded shaft coupling combined shear stress are as follows:

Metric units:

$$S_{CS} = (1 \times 10^3) \times \sqrt{\left(\frac{2 \times F_A}{\pi \times (D^2 - d^2)}\right)^2 + \left(\frac{4.86 \times 10^7 \times P}{n \times (D^3 - d^3)}\right)^2}$$

US customary units:

$$S_{CS} = \sqrt{\left(\frac{2 \times F_A}{\pi \times (D^2 - d^2)}\right)^2 + \left(\frac{321{,}000 \times P}{n \times (D^3 - d^3)}\right)^2}$$

Where:

S_{CS} = combined shear stress, in kPa (psi)

F_A = axial thrust acting through the shafting, in N (lb)

D = coupling outside diameter, in mm (in)

d = coupling inside diameter at the root of the threads, in mm (in)

P = power transmitted by the shaft, in kW (hp)

n = rotational speed of pump, in rpm

14.3.6.5.3 Stretch (axial elongation) in vertical pumps

Shaft stretch Is considered in the design of vertically suspended pumps, in particular, a semiopen impeller and close running clearance to the pump stationary casing or with long, deep-setting pumps with semiopen or enclosed impellers where shaft stretch can be significant. Shaft stretch is not normally considered in horizontal pumps.

The net axial downthrust force is carried by the pump shaft. The shaft will stretch, i.e., elongate, under this load. Before the pump starts up, any stretch that occurs is due to rotor weight, the sum of the static forces. The thrust load will increase after the pump starts up due to the addition of the dynamic forces. To prevent the impeller from bottoming against the bowl and damaging the pump when the shaft stretches under running load, the impeller axial position in the bowl must be set before the pump starts. Shaft elongation is maximized at shut-off flow.

On very long, deep-settings pumps, shaft stretch may exceed the axial clearance available within the bowl assembly. This requires special machining for increased clearance with some reduction in efficiency. Increasing the diameter of the lineshaft will reduce the axial stretch.

The allowable axial adjustment, the maximum axial clearance (endplay), with axial flow and semiopen impeller pumps is typically very small. These pumps require tight clearances to prevent large losses and reduction of pump performance.

Design axial endplay – lift setting > (Shaft stretch – Column stretch)

Shaft stretch

Metric units:

$$SS = 1000 \times \frac{K_t \times H_b \times L_C}{A_{sft} \times E_s}$$

US customary units:

$$SS = \frac{K_t \times H_b \times L_C}{A_{sft} \times E_s}$$

Where:

SS = shaft stretch, in mm (in)

K_t = thrust factor, in kg/m (lb/ft) (supplied by pump manufacturer)

H_b = bowl head, in m (ft)

L_c = column length, in m (ft)

A_{sft} = area, in mm^2 (in^2)

E_s = shaft modulus of elasticity, in mPa (psi)

Column stretch

Metric units:

$$CS = 1000 \times \frac{W_w \times L_c}{(A_{col} + A_{tube}) \times E_c}$$

US customary units:

$$CS = \frac{W_w \times L_c}{(A_{col} + A_{tube}) \times E_c}$$

Where:

C_S = column stretch, in mm (in)

W_w = weight of fluid in column above static water level in N (lb)

L_c = column length, in m (ft)

A_{col} = cross-sectional column area, in mm² (in²) 3.14 × (OD_{col}^{2-} ID_{col}^{2})/4

A_{tube} = cross-sectional enclosed tube area, in mm² (in²) (= 0 for open lineshaft pumps)
Ec = column modulus of elasticity, in mPa (psi)

14.3.6.5.4 Thermal expansion

For services above 121 °C (250 °F) with metallic and 66 °C (150 °F) for polymers, the design of components and axial setting of impellers has to account for the difference in thermal expansion between column and discharge pipe, and column and shaft. For deep-set pumps, axial setting impacts due to thermal expansion must always be considered as small temperature changes can result in large axial movement.

14.3.6.5.5 Method of calculating dry critical speed: open lineshaft vertical pumps

It is important that the lateral mode of shaft vibrations be considered when selecting shaft diameter and bearing spacing for both lineshaft and head shafts.

When the natural frequency of the shaft supported between two bearing bushings is calculated, it is often based on the assumption made for calculating the natural frequency of uniform beams. Usually, first and second critical frequency is checked. The lateral natural frequency, F_n, in rpm, is calculated per the following equations.

Metric units:

$$F_n = 0.762 \times \frac{N}{L} \times 83.034 \times \sqrt{\frac{g}{w_L}} \times \left[1.4504 \times 10^{-4} \times E \times I \times \left(7.9797 \times 10^{-2} \times \frac{N}{L}\right)^2 + 0.2249 \times F\right]^{0.5}$$

US customary units:

$$F_n = 30 \times \frac{N}{L} \times \sqrt{\frac{g}{w_L}} \times \left[E \times I \times \left(3.1416 \times \frac{N}{L}\right)^2 + F\right]^{0.5}$$

Where:

N = calculated critical frequency mode number; $N = 1$ for first mode, $N = 2$ for second mode

L = shaft length between bearing bushing supports, in m (in)

g = acceleration due to gravity, in m/s² (in/s²)

W_L = shaft weight per unit length, in N/m (lb/in)

E = modulus of elasticity of shaft material, in N/m^2 (lb/in^2)

I = moment of inertia of shaft's cross section, in mm^4 (in^4), $I = 3.1416 \times \frac{d^4}{64}$, d = shaft diameter, in mm (in)

F = axial force on shaft, in N (lb), equal to pump thrust plus weight of rotational components below the shaft location

Using the above equation for enclosed lineshaft natural frequency analysis gives uncertain results because of the enclosing-tube effect.

Most lineshaft bearing bushings are spaced at intervals not exceeding 3.0 m (10 ft). The critical frequency calculations for shaft sizes between 25 mm (1 in) and 40 mm (1.50 in) with 3.0-m (10-ft) spacing comes very close to the range of first critical. However, with good manufacturing practices in preparation of the pump drive-train parts, it has been found by practical experience that pumps of lineshaft, water-lubricated rubber bearing bushing construction at 3.0-m (10-ft) spacing operate successfully at 1800 rpm without excessive vibrations. At this same rotational speed, metal bearing bushing open lineshaft construction spacing is typically between 1.5 and 3.0 m (5 and 10 ft). Both enclosed and open lineshaft construction allow increased spacing with lower rpm. At 3000 or 3600 rpm, the bearing spacing has the smallest values. The manufacturer's suggested bearing spacing is to be used.

Appendix A

Introduction to general applications

Within the application categories described below, many services are not unique to one segment of industry. For this purpose, Appendix A is included to cover numerous aspects of general service applications and operational aspects that are commonly of interest across a variety of industry segments. Principal features of pumps and the necessary precautions for proper use are pointed out. Some elements of pump service are also covered by this category. These services cover a diverse range of commonly encountered pump application issues.

A.1 General service applications

A.1.1 Transfer pumping

Overview

This service is normally a part of a process where a fluid is transferred from one location or process to another in a given plant.

Services

There are many services that require transfer of fluid. Typically the requirement is for a low developed pressure but high flow rate.

Fluids handled

Almost all fluids may be encountered in this application. The limitations on fluid are determined mainly by the design and materials of construction of the pump used.

Pump types used

Almost all rotodynamic pumps are suitable and capable of operating in transfer service.

The most common pump types used in this type of application are single-stage overhung (OH) and single-stage between-bearings pumps (BB1 and BB2). Various types of vertically suspended pumps (VS) can also be used.

Special features

The type of pumps used will vary with the duty involved as well as construction details and materials of construction. Accurate specifications of the liquid characteristics and the range of suction pressures expected must be provided with rate of flow and total head for the pump manufacturer to make a proper selection.

Pumps used in the transfer of fluids from tanks need to be designed for the inlet conditions that occur when the fluid in the tank is at its lowest level. To prevent the pump from losing prime, adequate control systems need to be in place to prevent drawdown of liquid beyond the safe design point of the pump.

A.1.2 Booster service

Overview

Rotodynamic pumps in this service handle liquids piped to them at various levels of pressure, normally above atmospheric, and discharge at a higher pressure into the system.