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The statistical analysis of fatigue data using the Weibull distribution

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The preparation of this Data Item

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THE STATISTICAL ANALYSIS OF FATIGUE DATA USING THE WEIBULL DISTRIBUTION

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THE STATISTICAL ANALYSIS OF FATIGUE DATA USING THE WEIBULL DISTRIBUTION

1. INTRODUCTION

This Data Item deals with and gives guidance on the use of the Weibull distribution for the analysis of engineering data and for the prediction of fatigue failure analysis. This Data Item accompanies ESDU 91041* and ESDU 92040†.

Understanding the topic of reliability is essential in all manufacturing industries. The reliability of a part is a measure of its quality depending on the life to the first failure or on the life between subsequent failures, for example. Note that if a part cannot be repaired, then its life to the first failure is also its final life. The failure life of a part can be any positive number and the distribution of failure lives is continuous and described by measures such as the probability density function (PDF) or its cumulative distribution function (CDF), its reliability function and its hazard function - see Section 2 for further definitions of these functions. For failure of a part due to fatigue processes, the relevant variable is number of load cycles applied rather than life in service (measured in hours or days), although both service life and number of fatigue cycles may be used to carry out a reliability analysis.

Statistical distributions are fitted to empirical data derived from either laboratory fatigue tests or from in-service reporting to estimate properties such as the reliability, the probability of failure at a specified time and the mean life of a component. The Weibull distribution (see Derivations 1 and 2), and it use for examining the statistical distribution of given fatigue data, is discussed in Section 3. As well as predicting fatigue life and related confidence limits around this data, the Weibull distribution can be used to analyse the probability of survival of a component subject to fatigue loading in-service or during testing.

Additionally, due to a volume effect (the increase in probability of identifying a critical defect in a greater volume of highly stressed material), the Weibull weakest-link theory can be used to show that the survival probability of a larger structure is less than that for smaller structures under the same operating conditions. A discussion of this and a brief outline of other uses of the Weibull distribution in fatigue life analysis is given in Section 4. In Section 5, an examination of the sources of uncertainty found when using statistical distributions is given. Two worked examples using the Weibull distribution to determine the fatigue life of a part are presented in Section 7.

1.1 Notation

c	constant (in Equation (3.14))
c'	constant (in Equation (3.15))
$\mathrm{d}a/\mathrm{d}N$	fatigue crack propagation rate
erf, erfc	error function, complementary error function
F	cumulative distribution function
F_i	cumulative distribution function of the ith failure

* ESDU 91041, 'The statistical analysis of data from normal distributions, with particular reference to small samples'.
† ESDU 92040, 'An introduction to the statistical analysis of engineering data'.

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f	probability density function
h	hazard function
j	position of a number in an array
$K(n)$	value for calculating the confidence interval for a given sample size
MR	median rank
m	constant
$\mathrm{N_o}$	mean life, or mean number of cycles, to failure
$\mathrm{N_R}$	reliability life, or number of cycles for a given reliability
$\mathrm{N_{1\text{-}P}}$	probability of failure
n	sample size, number of observations
P	probability
R	reliability function
s	sample variance
t	time to failure
t_i	time to failure of the ith failure
X	constant
X	variable with a normal distribution
x	a value of X
$\bar{x}$	mean value of x
x_i	ith value of x
x_R	reliable life
Y	constant, see Section 3.2
Y	function, see Section A2
$Z_{a/2}$	critical value for a given confidence interval
Z	variable with a standard normal distribution
z	a value of Z
β	shape parameter