Designation: E1876 – 15

Standard Test Method for Dynamic Young's Modulus, Shear Modulus, and Poisson's Ratio by Impulse Excitation of Vibration[1]

This standard is issued under the fixed designation E1876; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This test method covers determination of the dynamic elastic properties of elastic materials at ambient temperatures. Specimens of these materials possess specific mechanical resonant frequencies that are determined by the elastic modulus, mass, and geometry of the test specimen. The dynamic elastic properties of a material can therefore be computed if the geometry, mass, and mechanical resonant frequencies of a suitable (rectangular or cylindrical geometry) test specimen of that material can be measured. Dynamic Young's modulus is determined using the resonant frequency in either the flexural or longitudinal mode of vibration. The dynamic shear modulus, or modulus of rigidity, is found using torsional resonant vibrations. Dynamic Young's modulus and dynamic shear modulus are used to compute Poisson's ratio.

1.2 Although not specifically described herein, this test method can also be performed at cryogenic and high temperatures with suitable equipment modifications and appropriate modifications to the calculations to compensate for thermal expansion.

1.3 There are material specific ASTM standards that cover the determination of resonance frequencies and elastic properties of specific materials by sonic resonance or by impulse excitation of vibration. Test Methods C215, C623, C747, C848, C1198, and C1259 may differ from this test method in several areas (for example; sample size, dimensional tolerances, sample preparation). The testing of these materials shall be done in compliance with these material specific standards. Where possible, the procedures, sample specifications and calculations are consistent with these test methods.

1.4 The values stated in SI units are to be regarded as standard. No other units of measurement are included in this standard.

1.5 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*[2]

C215 Test Method for Fundamental Transverse, Longitudinal, and Torsional Resonant Frequencies of Concrete Specimens
C372 Test Method for Linear Thermal Expansion of Porcelain Enamel and Glaze Frits and Fired Ceramic Whiteware Products by the Dilatometer Method
C623 Test Method for Young's Modulus, Shear Modulus, and Poisson's Ratio for Glass and Glass-Ceramics by Resonance
C747 Test Method for Moduli of Elasticity and Fundamental Frequencies of Carbon and Graphite Materials by Sonic Resonance
C848 Test Method for Young's Modulus, Shear Modulus, and Poisson's Ratio For Ceramic Whitewares by Resonance
C1161 Test Method for Flexural Strength of Advanced Ceramics at Ambient Temperature
C1198 Test Method for Dynamic Young's Modulus, Shear Modulus, and Poisson's Ratio for Advanced Ceramics by Sonic Resonance
C1259 Test Method for Dynamic Young's Modulus, Shear Modulus, and Poisson's Ratio for Advanced Ceramics by Impulse Excitation of Vibration
E6 Terminology Relating to Methods of Mechanical Testing
E177 Practice for Use of the Terms Precision and Bias in ASTM Test Methods

3. Terminology

3.1 *Definitions:*

3.1.1 The definitions of terms relating to mechanical testing appearing in Terminology E6 and C1198 should be considered as applying to the terms used in this test method.

[1] This test method is under the jurisdiction of ASTM Committee E28 on Mechanical Testing and is the direct responsibility of Subcommittee E28.04 on Uniaxial Testing.

Current edition approved Dec. 15, 2015. Published March 2016. Originally approved in 1997. Last previous edition approved in 2009 as E1876 – 09. DOI: 10.1520/E1876-15.

[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

3.1.2 *dynamic elastic modulus, n*—the elastic modulus, either Young's modulus or shear modulus, that is measured in a dynamic mechanical measurement.

3.1.3 *dynamic mechanical measurement, n*—a technique in which either the modulus or damping, or both, of a substance under oscillatory applied force or displacement is measured as a function of temperature, frequency, or time, or combination thereof.

3.1.4 *elastic limit* [FL^{-2}], *n*—the greatest stress that a material is capable of sustaining without permanent strain remaining upon complete release of the stress. **E6**

3.1.5 *modulus of elasticity* [FL^{-2}], *n*—the ratio of stress to corresponding strain below the proportional limit.

3.1.5.1 *Discussion*—The stress-strain relationships of many materials do not conform to Hooke's law throughout the elastic range, but deviate therefrom even at stresses well below the elastic limit. For such materials, the slope of either the tangent to the stress-strain curve at the origin or at a low stress, the secant drawn from the origin to any specified point on the stress-strain curve, or the chord connecting any two specified points on the stress-strain curve is usually taken to be the "modulus of elasticity." In these cases, the modulus should be designated as the "tangent modulus," the "secant modulus," or the "chord modulus," and the point or points on the stress-strain curve described. Thus, for materials where the stress-strain relationship is curvilinear rather than linear, one of the four following terms may be used:

(a) initial tangent modulus [FL^{-2}], *n*—the slope of the stress-strain curve at the origin.

(b) tangent modulus [FL^{-2}], *n*—the slope of the stress-strain curve at any specified stress or strain.

(c) secant modulus [FL^{-2}], *n*—the slope of the secant drawn from the origin to any specified point on the stress-strain curve.

(d) chord modulus [FL^{-2}], *n*—the slope of the chord drawn between any two specified points on the stress-strain curve below the elastic limit of the material.

3.1.5.2 *Discussion*—Modulus of elasticity, like stress, is expressed in force per unit of area (pounds per square inch, etc.).

3.1.6 *Poisson's ratio,* μ, *n*—the negative of the ratio of transverse strain to the corresponding axial strain resulting from an axial stress below the proportional limit of the material.

3.1.6.1 *Discussion*—Poisson's ratio may be negative for some materials, for example, a tensile transverse strain will result from a tensile axial strain.

3.1.6.2 *Discussion*—Poisson's ratio will have more than one value if the material is not isotropic. **E6**

3.1.7 *proportional limit* [FL^{-2}] , *n*—the greatest stress that a material is capable of sustaining without deviation from proportionality of stress to strain (Hooke's law). **E6**

3.1.7.1 *Discussion*—Many experiments have shown that values observed for the proportional limit vary greatly with the sensitivity and accuracy of the testing equipment, eccentricity of loading, the scale to which the stress-strain diagram is plotted, and other factors. When determination of proportional limit is required, the procedure and the sensitivity of the test equipment should be specified.

3.1.8 *shear modulus, G* [FL^{-2}], *n*—the ratio of shear stress to corresponding shear strain below the proportional limit, also called *torsional modulus* and *modulus of rigidity*.

3.1.8.1 *Discussion*—The value of the shear modulus may depend on the direction in which it is measured if the material is not isotropic. Wood, many plastics and certain metals are markedly anisotropic. Deviations from isotropy should be suspected if the shear modulus differs from that determined by substituting independently measured values of Young's modulus, *E*, and Poisson's ratio, μ, in the relation:

$$G = \frac{E}{2(1+\mu)}$$

3.1.8.2 *Discussion*—In general, it is advisable in reporting values of shear modulus to state the range of stress over which it is measured. **E6**

3.1.9 *Young's modulus, E* [FL^{-2}], *n*—the ratio of tensile or compressive stress to corresponding strain below the proportional limit of the material. **E6**

3.2 *Definitions of Terms Specific to This Standard:*

3.2.1 *anti-nodes, n*—two or more locations in an unconstrained slender rod or bar in resonance that have local maximum displacements.

3.2.1.1 *Discussion*—For the fundamental flexure resonance, the anti-nodes are located at the two ends and the center of the specimen.

3.2.2 *elastic, adj*—the property of a material such that an application of stress within the elastic limit of that material making up the body being stressed will cause an instantaneous and uniform deformation, which will be eliminated upon removal of the stress, with the body returning instantly to its original size and shape without energy loss. Most elastic materials conform to this definition well enough to make this resonance test valid.

3.2.3 *flexural vibrations, n*—the vibrations that occur when the oscillations in a slender rod or bar are in a plane normal to the length dimension.

3.2.4 *homogeneous, adj*—the condition of a specimen such that the composition and density are uniform, so that any smaller specimen taken from the original is representative of the whole.

3.2.4.1 *Discussion*—Practically, as long as the geometrical dimensions of the test specimen are large with respect to the size of individual grains, crystals, components, pores, or microcracks, the body can be considered homogeneous.

3.2.5 *in-plane flexure, n*—for rectangular parallelepiped geometries, a flexure mode in which the direction of displacement is in the major plane of the test specimen.

3.2.6 *isotropic, adj*—the condition of a specimen such that the values of the elastic properties are the same in all directions in the material.

3.2.6.1 *Discussion*—Materials are considered isotropic on a macroscopic scale, if they are homogeneous and there is a random distribution and orientation of phases, crystallites, components, pores, or microcracks.

3.2.7 *longitudinal vibrations, n*—the vibrations that occur when the oscillations in a slender rod or bar are parallel to the length of the rod or bar.

3.2.8 *nodes, n*—one or more locations of a slender rod or bar in resonance that have a constant zero displacement.

3.2.8.1 *Discussion*—For the fundamental flexural resonance, the nodes are located at 0.224 L from each end, where L is the length of the specimen.

3.2.9 *out-of-plane flexure, n*—for rectangular parallelepiped geometries, a flexure mode in which the direction of displacement is perpendicular to the major plane of the test specimen.

3.2.10 *resonant frequency, n*—naturally occurring frequencies of a body driven into flexural, torsional, or longitudinal vibration that are determined by the elastic modulus, mass, and dimensions of the body.

3.2.10.1 *Discussion*—The lowest resonant frequency in a given vibrational mode is the fundamental resonant frequency of that mode.

3.2.11 *slender rod or bar, n*—in dynamic elastic property testing, a specimen whose ratio of length to minimum cross-sectional dimension is at least five and preferably in the range from 20 to 25.

3.2.12 *torsional vibrations, n*—the vibrations that occur when the oscillations in each cross-sectional plane of a slender rod or bar are such that the plane twists around the length dimension axis.

3.3 *Symbols:*

A = plate constant; used in Eq A1.1
D = diameter of rod or diameter of disk
D_e = effective diameter of the bar; defined in Eq 10 and Eq 11
E = dynamic Young's modulus; defined in Eq 1 and Eq 4, and Eq A1.4
E_1 = first natural calculation of the dynamic Young's modulus, used in Eq A1.2
E_2 = second natural calculation of the dynamic Young's modulus. used in Eq A1.3
G = dynamic shear modulus, defined in Eq 12, Eq 14, and Eq A1.5
K = correction factor for the fundamental longitudinal mode to account for the finite diameter-to-length ratio and Poisson's Ratio, defined in Eq 8
K_i = geometric factor for the resonant frequency of order i, see Table A1.2 and Table A1.3
L = specimen length
M_T = dynamic elastic modulus at temperature T (either the dynamic Young's modulus E, or the dynamic shear modulus G)
M_0 = dynamic elastic modulus at room temperature (either the dynamic Young's modulus E or the dynamic shear modulus G)
R = correction factor the geometry of the bar, defined in Eq 13
T_1 = correction factor for fundamental flexural mode to account for finite thickness of bar and Poisson's ratio; defined in Eq 2
T_1' = correction factor for fundamental flexural mode to account for finite diameter of rod, Poisson's ratio; defined in Eq 4 and Eq 6
b = specimen width
f = frequency
f_0 = resonant frequency at room temperature in furnace or cryogenic chamber
f_1 = first natural resonant frequency; used in Eq A1.2
f_2 = second natural frequency; used in Eq A1.3
f_f = fundamental resonant frequency of bar in flexure; used in Eq 1
f_l = fundamental longitudinal resonant frequency of a slender bar; used in Eq 7 and Eq 9
f_T = resonant frequency measured in the furnace or cryogenic chamber at temperature T, used in Eq 16
f_t = fundamental resonant frequency of bar in torsion; used in Eq 12 and Eq 14
m = specimen mass
n = the order of the resonance (n=1,2,3,...)
r = radius of the disk, used in Eq A1.1
t = specimen, disk or bar, thickness
T_1 = correction factor for fundamental flexural mode to account for finite thickness of the bar and Poisson's ratio; defined in Eq 2
T'_1 = correction factor for fundamental flexural mode to account for finite thickness of the rod and Poisson's ratio; defined in Eq 4
ΔT = temperature difference between the test temperature T and room temperature, used in Eq 16
α = average linear thermal expansion coefficient (mm/mm/°C) from room temperature to test temperature; used in Eq 16
μ = Poisson's ratio
ρ = density of the disk; used in Eq A1.1

4. Summary of Test Method

4.1 This test method measures the fundamental resonant frequency of test specimens of suitable geometry by exciting them mechanically by a singular elastic strike with an impulse tool. A transducer (for example, contact accelerometer or non-contacting microphone) senses the resulting mechanical vibrations of the specimen and transforms them into electric signals. Specimen supports, impulse locations, and signal pick-up points are selected to induce and measure specific modes of the transient vibrations. The signals are analyzed, and the fundamental resonant frequency is isolated and measured by the signal analyzer, which provides a numerical reading that is (or is proportional to) either the frequency or the period of the specimen vibration. The appropriate fundamental resonant frequencies, dimensions, and mass of the specimen are used to calculate dynamic Young's modulus, dynamic shear modulus, and Poisson's ratio.

5. Significance and Use

5.1 This test method may be used for material development, characterization, design data generation, and quality control purposes.

5.2 This test method is specifically appropriate for determining the dynamic elastic modulus of materials that are elastic, homogeneous, and isotropic (**1**).[3]

5.3 This test method addresses the room temperature determination of dynamic elastic moduli of elasticity of slender bars (rectangular cross section) rods (cylindrical), and flat disks. Flat plates may also be measured similarly, but the required equations for determining the moduli are not presented.

5.4 This dynamic test method has several advantages and differences from static loading techniques and from resonant techniques requiring continuous excitation.

5.4.1 The test method is nondestructive in nature and can be used for specimens prepared for other tests. The specimens are subjected to minute strains; hence, the moduli are measured at or near the origin of the stress-strain curve, with the minimum possibility of fracture.

5.4.2 The impulse excitation test uses an impact tool and simple supports for the test specimen. There is no requirement for complex support systems that require elaborate setup or alignment.

5.5 This technique can be used to measure resonant frequencies alone for the purposes of quality control and acceptance of test specimens of both regular and complex shapes. A range of acceptable resonant frequencies is determined for a specimen with a particular geometry and mass. The technique is particularly suitable for testing specimens with complex geometries (other than parallelepipeds, cylinders/rods, or disks) that would not be suitable for testing by other procedures. Any specimen with a frequency response falling outside the prescribed frequency range is rejected. The actual dynamic elastic modulus of each specimen need not be determined as long as the limits of the selected frequency range are known to include the resonant frequency that the specimen must possess if its geometry and mass are within specified tolerances.

5.6 If a thermal treatment or an environmental exposure affects the elastic response of the test specimen, this test method may be suitable for the determination of specific effects of thermal history, environment exposure, and so forth. Specimen descriptions should include any specific thermal treatments or environmental exposures that the specimens have received.

6. Interferences

6.1 The relationships between resonant frequency and dynamic elastic modulus presented herein are specifically applicable to homogeneous, elastic, isotropic materials.

6.1.1 This method of determining the moduli is applicable to composite and inhomogeneous materials only with careful consideration of the effect of inhomogeneities and anisotropy. The character (volume fraction, size, morphology, distribution, orientation, elastic properties, and interfacial bonding) of the reinforcement and inhomogeneities in the specimens will have a direct effect on the elastic properties of the specimen as a whole. These effects must be considered in interpreting the test results for composites and inhomogeneous materials.

6.1.2 The procedure involves measuring transient elastic vibrations. Materials with very high damping capacity may be difficult to measure with this technique if the vibration damps out before the frequency counter can measure the signal (commonly within three to five cycles).

6.1.3 If specific surface treatments (coatings, machining, grinding, etching, and so forth) change the elastic properties of the near-surface material, there will be accentuated effects on the properties measured by this flexural method, as compared to static/bulk measurements by tensile or compression testing.

6.1.4 This test method is not satisfactory for specimens that have major discontinuities, such as large cracks (internal or surface) or voids.

6.2 This test method for determining moduli is limited to specimens with regular geometries (rectangular parallelepiped, cylinders, and disks) for which analytical equations are available to relate geometry, mass, and modulus to the resonant vibration frequencies. This test method is not appropriate for determining the elastic properties of materials that cannot be fabricated into such geometries.

6.2.1 The analytical equations assume parallel and concentric dimensions for the regular geometries of the specimen. Deviations from the specified tolerances for the dimensions of the specimens will change the resonant frequencies and introduce error into the calculations.

6.2.2 Edge treatments such as chamfers or radii are not considered in the analytical equations. Edge chamfers change the resonant frequency of the test bars and introduce error into the calculations of the dynamic elastic modulus. It is recommended that specimens for this test method not have chamfered or rounded edges.

6.2.3 For specimens with as-fabricated and rough or uneven surfaces, variations in dimension can have a significant effect in the calculations. For example, in the calculation of dynamic elastic modulus, the modulus value is inversely proportional to the cube of the thickness. Uniform specimen dimensions and precise measurements are essential for accurate results.

6.3 This test method assumes that the specimen is vibrating freely, with no significant restraint or impediment. Specimen supports should be designed and located properly in accordance with the instructions so the specimen can vibrate freely in the desired mode. In using direct contact transducers, the transducer should be positioned away from anti-nodes and with minimal force to avoid interference with free vibration.

6.4 Proper location to the impulse point and transducer is important in introducing and measuring the desired vibration mode. The locations of the impulse point and transducer should not be changed in multiple readings; changes in position may develop and detect alternate vibration modes. In the same manner, the force used in impacting should be consistent in multiple readings.

6.5 If the frequency readings are not repeatable for a specific set of impulse and transducer locations on a specimen, it may be because several different modes of vibration are being developed and detected in the test. The geometry of the

[3] The boldface numbers in parentheses refer to the list of references at the end of this standard.