Paragraph 4.15.3.5,

g) Maximum circumferential bending moment - the distribution of the circumferential bending moment at the saddle support is dependent on the use of stiffeners at the saddle location. For a cylindrical shell without a stiffening ring, the maximum circumferential bending moment is shown in Figure 4.15.6 Sketch (a) and is calculated as follows.

$$M_{\beta} = K_7 Q R_m = (0.0504)(50459.0)(31.5625) = 80267.7 \; in-lbs$$

Where the coefficient K_7 is found in Table 4.15.1,

When $\dfrac{a}{R_m} \geq 1.0$, $K_7 = K_6$

$$\left\{ \frac{a}{R_m} = \frac{41.0}{31.5625} = 1.2990 \right\} \geq 1.0 \rightarrow K_7 = K_6 = 0.0504$$

$$K_6 = \left(\frac{\dfrac{3\cos\beta}{4}\left(\dfrac{\sin\beta}{\beta}\right)^2 - \dfrac{5\sin\beta\cos^2\beta}{4\beta} + \dfrac{\cos^3\beta}{2} - \dfrac{\sin\beta}{4\beta} + \dfrac{\cos\beta}{4} - \beta\sin\beta\left[\left(\dfrac{\sin\beta}{\beta}\right)^2 - \dfrac{1}{2} - \dfrac{\sin 2\beta}{4\beta}\right]}{2\pi\left[\left(\dfrac{\sin\beta}{\beta}\right)^2 - \dfrac{1}{2} - \dfrac{\sin 2\beta}{4\beta}\right]} \right)$$

$$K_6 = \left(\frac{\dfrac{3\cos[2.0682]}{4}\left(\dfrac{\sin[2.0682]}{2.0682}\right)^2 - \dfrac{5\sin[2.0682]\cos^2[2.0682]}{4(2.0682)} + \dfrac{\cos^3[2.0682]}{2} - \dfrac{\sin[2.0682]}{4(2.0682)} + \dfrac{\cos[2.0682]}{4} - (2.0682)\sin[2.0682]\left[\left(\dfrac{\sin[2.0682]}{2.0682}\right)^2 - \dfrac{1}{2} - \left(\dfrac{\sin[2(2.0682)]}{4(2.0682)}\right)\right]}{2\pi\left[\left(\dfrac{\sin[2.0682]}{2.0682}\right)^2 - \dfrac{1}{2} - \left(\dfrac{\sin[2(2.0682)]}{4(2.0682)}\right)\right]} \right) = 0.0504$$

$$\beta = \pi - \frac{\theta}{2} = \pi - \frac{123.0\left(\frac{\pi}{180}\right)}{2} = 2.0682\ rad$$

h) Width of cylindrical shell - the width of the cylindrical shell that contributes to the strength of the cylindrical shell at the saddle location shall be determined as follows.

$$x_1, x_2 \leq 0.78\sqrt{R_m t} = 0.78\sqrt{31.5625(2.875)} = 7.4302\ in$$

If the width $(0.5b + x_1)$ extends beyond the limit of a, as shown in Figure 4.15.2, then the width x_1 shall be reduced such as not to exceed a.

$$\{(0.5b + x_1) = 0.5(8.0) + 7.4302 = 11.4302\ in\} \leq \{a = 41.0\ in\} \qquad \textit{Satisfied}$$

i) Circumferential stresses in the cylindrical shell without stiffening ring(s).

1) The maximum compressive circumferential membrane stress in the cylindrical shell at the base of the saddle support shall be calculated as follows.

$$\sigma_6 = \frac{-K_5 Q k}{t(b + x_1 + x_2)} = \frac{-0.7492(50459.0)(0.1)}{2.875(8.0 + 7.4302 + 7.4302)} = -57.5\ psi$$

Where the coefficient K_5 is found in Table 4.15.1,

$$K_5 = \frac{1 + \cos\alpha}{\pi - \alpha + \sin\alpha \cdot \cos\alpha} = \frac{1 + \cos[1.9648]}{\pi - (1.9648) + \sin[1.9648] \cdot \cos[1.9648]} = 0.7492$$

$$k = 0.1 \qquad \textit{when the vessel is welded to the saddle support}$$

2) The circumferential compressive membrane plus bending stress at Points G and H of Figure 4.15.6 Sketch (a) is determined as follows.

iii) If $L \geq 8R_m$, then the circumferential compressive membrane plus bending stress shall be computed using Equation (4.15.24).

Since $\{L = 292.0\ in\} \geq \{8R_m = 8(31.5625) = 252.5\ in\}$, the criterion is satisfied.

$$\sigma_7 = \frac{-Q}{4t(b + x_1 + x_2)} - \frac{3K_7 Q}{2t^2}$$

$$\sigma_7 = \frac{-(50459.0)}{4(2.875)(8 + 7.4302 + 7.4302)} - \frac{3(0.0504)(50459.0)}{2(2.875)^2} = -653.4\ psi$$

3) The stresses at σ_6 and σ_7 may be reduced by adding a reinforcement or wear plate at the saddle location that is welded to the cylindrical shell.

A wear plate was not specified in this example.

j) Circumferential stress in the cylindrical shell with a stiffening ring along the plane of the saddle support.

Stiffeners were not specified in the example.

k) Circumferential stress in the cylindrical shell with stiffening rings on both sides of the saddle support.

Stiffeners were not specified in the example.

l) Acceptance Criteria:

$$\left\{ |\sigma_6| = |57.5|\ psi \right\} \le \left\{ S = 23500\ psi \right\} \qquad True$$

$$\left\{ |\sigma_7| = |653.4|\ psi \right\} \le \left\{ 1.25S = 1.25(23500) = 29375\ psi \right\} \qquad True$$

Paragraph 4.15.3.6,

The horizontal force at the minimum section at the low point of the saddle is given by Equation (4.15.42). The saddle shall be designed to resist this force.

$$F_h = Q\left(\frac{1+\cos\beta - 0.5\sin^2\beta}{\pi - \beta + \sin\beta\cdot\cos\beta} \right)$$

$$F_h = (50459.0)\left(\frac{1+\cos[2.0682] - 0.5\sin^2[2.0682]}{\pi - (2.0682) + \sin[2.0682]\cdot\cos[2.0682]} \right) = 10545.1\ lbs$$

Note: The horizontal splitting force is equal to the sum of all of the horizontal reactions at the saddle due to the weight loading of the vessel. The splitting force is used to calculate tension stress and bending stress in the web of the saddle. The following provides one possible method of calculating the tension and bending stress in the web and its acceptance criteria. However, other methods may also be deemed acceptable by the Manufacturer and accepted by the Authorized Inspector.

The membrane stress is given by,

$$\left\{ \sigma_t = \frac{F_h}{A_s} \right\} \le \left\{ 0.6S_y \right\}$$

where A_s is the cross-sectional area of the web at the low point of the saddle with units of in^2, and S_y is the yield stress of the saddle material with units of psi.

The bending stress is given by,

$$\left\{ \sigma_b = \frac{F_h\cdot d\cdot c}{I} \right\} \le \left\{ 0.66S_y \right\}$$

where d is the moment arm of the horizontal splitting force, measured from the center of gravity of the saddle arc to the bottom of the saddle baseplate with units of in, c is the distance from the centroid of the saddle composite section to the extreme fiber with units of in, I is the moment of inertia of the composite section of the saddle with units of in^4, and S_y is the yield stress of the saddle material with units of psi.

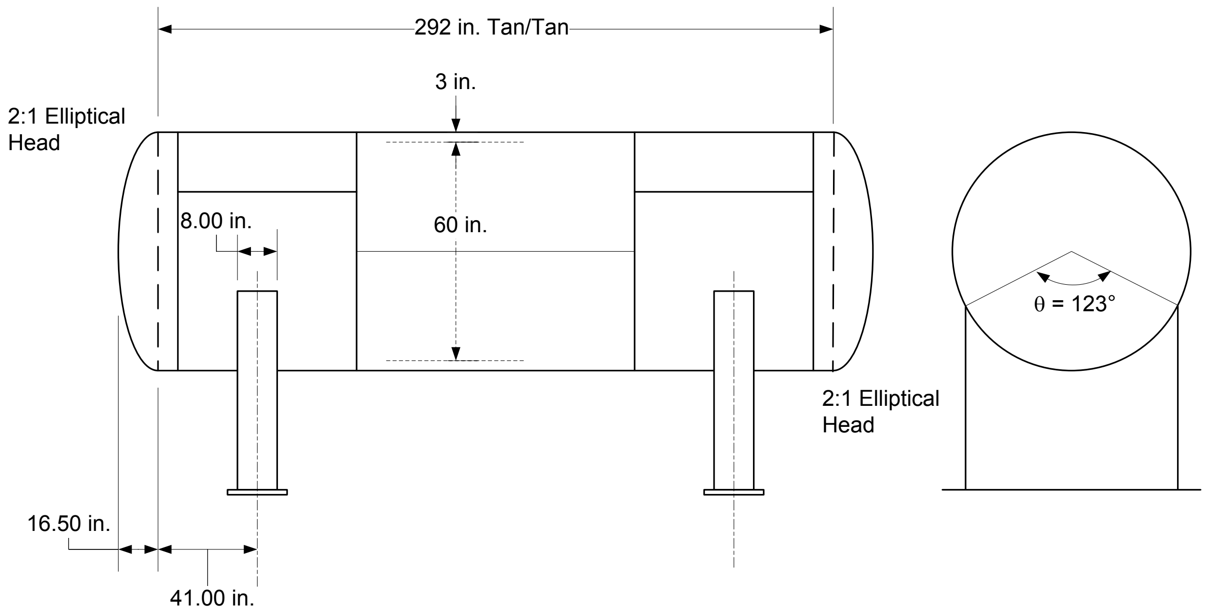

Figure E4.15.1 - Saddle Details

4.15.2 Example E4.15.2 – Vertical Vessel, Skirt Design

Determine if the proposed cylindrical vessel skirt is adequately designed considering the following loading conditions.

Skirt Data:

- Material = $SA-516,\ Grade\ 70$
- Design Temperature = $300°F$
- Skirt Inside Diameter = $150.0\ in$
- Thickness = $0.625\ in$
- Length of Skirt = $147.0\ in$
- Allowable Stress at Design Temperature = $22400\ psi$
- Modulus of Elasticity at Design Temperature = $28.3E+06\ psi$
- Yield Strength at Design Temperature = $33600\ psi$
- Axial Force, Weight = $-363500\ lbs$
- Axial Force, Appurtenance Live Loading = $-85700\ lbs$
- Bending Moment, Appurtenance Loading = $90580\ in-lbs$
- Bending Moment, Earthquake Loading = $18550000\ in-lbs$
- Bending Moment, Wind Loading = $29110000\ in-lbs$

Adjust variable for corrosion and determine outside dimensions.

$$D = 150.0 + 2(Corrosion\ Allowance) = 150.0 + 2(0.0) = 150.0\ in$$

$$R = 0.5D = 0.5(150.0) = 75.0\ in$$

$$t = 0.625 - Corosion\ Allowance = 0.625 - 0.0 = 0.625\ in$$

$$D_o = 150.0 + 2(Uncorroded\ Thickness) = 150.0 + 2(0.625) = 151.25\ in$$

$$R_o = 0.5D_o = 0.5(151.25) = 75.625\ in$$

Evaluate per paragraph 4.15.4 with reference to paragraph 4.3.10.

The loads transmitted to the base of the skirt are given in the Table E4.15.2.2. Note that this table is given in terms of the load parameters and load combinations shown in Table 4.1.1 and Table 4.1.2. (Table E4.15.2.1 of this example). As shown in Table E4.15.2.1, the acceptance criteria is that the general primary membrane stress for each load case must be less than or equal to the allowable stress at the specified design condition.

In accordance with paragraph 4.3.10.2, the following procedure shall be used to design cylindrical, spherical, and conical shells subjected to internal pressure plus supplemental loads of applied net section axial force, bending moment, and torsional moment. By inspection of the results shown in Table E4.15.2.3, Load Case 5 is determined to be a potential governing load case. The pressure, net section axial force, and bending moment at the location of interest for Load Case 5 are:

$$0.9P + P_s = 0.0 \ psi$$
$$F_5 = -363500 \ lbs$$
$$M_5 = 17466000 \ in-lbs$$
$$M_{t5} = 0.0 \ in-lbs$$

Determine applicability of the rules of paragraph 4.3.10 based on satisfaction of the following requirements.

The section of interest is at least $2.5\sqrt{Rt}$ away from any major structural discontinuity.

$$2.5\sqrt{Rt} = 2.5\sqrt{(75.0)(0.625)} = 17.1163 \ in$$

Shear force is not applicable.

The shell R/t ratio is greater than 3.0, or:

$$\left\{R/t = \frac{75.0}{0.625} = 120.0\right\} > 3.0 \qquad \qquad True$$

a) STEP 1 – Calculate the membrane stress. For the skirt, weld joint efficiency is set as $E = 1.0$. Note, that the maximum bending stress occurs at $\theta = 0.0 \ deg$.

$$\sigma_{\theta m} = \frac{P}{E(D_o - D)} = \frac{P}{E(151.25 - 150.0)} = 0.0 \ psi$$

$$\sigma_{sm} = \frac{1}{E}\left(\frac{PD^2}{D_o^2 - D^2} + \frac{4F}{\pi\left(D_o^2 - D^2\right)} \pm \frac{32MD_o\cos[\theta]}{\pi\left(D_o^4 - D^4\right)}\right)$$

$$\sigma_{sm} = \frac{1}{1.0}\left(0.0 + \frac{4(-363500)}{\pi\left((151.25)^2 - (150.0)^2\right)} \pm \frac{32(17466000)(151.25)\cos[0.0]}{\pi\left((151.25)^4 - (150.0)^4\right)}\right)$$

$$\sigma_{sm} = \begin{Bmatrix} -1229.0724 + 1574.7814 = 345.7090 \ psi \\ -1229.0724 - 1574.7814 = -2803.8538 \ psi \end{Bmatrix}$$

$$\tau = \frac{16M_tD_o}{\pi\left(D_o^4 - D^4\right)} = 0.0 \ psi$$

b) STEP 2 – Calculate the principal stresses.

$$\sigma_1 = 0.5\left(\sigma_{\theta m} + \sigma_{sm} + \sqrt{\left(\sigma_{\theta m} - \sigma_{sm}\right)^2 + 4(\tau)^2}\right)$$

$$\sigma_1 = \begin{Bmatrix} 0.5\left(0.0 + 345.7090 + \sqrt{(0.0 - 345.7090)^2 + 4(0.0)^2}\right) = 345.7090 \ psi \\ 0.5\left(0.0 + (-2803.8538) + \sqrt{(0.0 - (-2803.8538))^2 + 4(0.0)^2}\right) = 0.0 \ psi \end{Bmatrix}$$

$$\sigma_2 = 0.5\left(\sigma_{\theta m} + \sigma_{sm} - \sqrt{\left(\sigma_{\theta m} - \sigma_{sm}\right)^2 + 4\left(\tau\right)^2}\right)$$

$$\sigma_2 = \begin{Bmatrix} 0.5\left(0.0 + 345.7090 - \sqrt{\left(0.0 - 345.7090\right)^2 + 4\left(0.0\right)^2}\right) = 0.0\ psi \\ 0.5\left(0.0 + \left(-2803.8538\right) - \sqrt{\left(0.0 - \left(-2803.8538\right)\right)^2 + 4\left(0.0\right)^2}\right) = -2803.8538\ psi \end{Bmatrix}$$

$$\sigma_3 = \sigma_r = -0.5P = 0.0\ psi$$

c) STEP 3 – Check the allowable stress acceptance criteria.

$$\sigma_e = \frac{1}{\sqrt{2}}\left[\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2\right]^{0.5} \le SE$$

$$\sigma_e = \begin{Bmatrix} \frac{1}{\sqrt{2}}\begin{bmatrix} \left(345.7090 - 0.0\right)^2 + \\ \left(0.0 - 0.0\right)^2 + \left(0.0 - 345.7090\right)^2 \end{bmatrix}^{0.5} = 345.7090\ psi \\ \frac{1}{\sqrt{2}}\begin{bmatrix} \left(0.0 - \left(-2803.8538\right)\right)^2 + \\ \left(\left(-2803.8538\right) - 0.0\right)^2 + \left(0.0 - 0.0\right)^2 \end{bmatrix}^{0.5} = 2803.8538\ psi \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_e = 345.7\ psi \\ \sigma_e = 2803.9\ psi \end{Bmatrix} \le \left\{SE = 22400\ psi\right\}$$

Since the equivalent stress is less than the acceptance criteria, the shell section is adequately designed considering Load Case 5.

d) STEP 4 – For cylindrical and conical shells, if the axial membrane stress, σ_{sm} is compressive, then Equation (4.3.45) shall be satisfied where F_{xa} is evaluated using paragraph 4.4.12.2 with $\lambda = 0.15$.

$$\sigma_{sm} \le F_{xa}$$

Since σ_{sm} is compressive, $\left\{\sigma_{sm} = -2803.8538\ psi < 0\right\}$, a buckling check is required.

VIII-2, paragraph 4.4.12.2.b – Axial Compressive Stress Acting Alone.

In accordance with paragraph 4.4.12.2.b, the value of F_{xa} is calculated as follows, with $\lambda = 0.15$.

The design factor FS used in paragraph 4.4.12.2.b is dependent on the predicted buckling stress F_{ic} and the material's yield strength, S_y as shown in paragraph 4.4.2. An initial calculation is required to determine the value of F_{xa} by setting $FS = 1.0$, with $F_{ic} = F_{xa}$. The initial value of F_{ic} is then compared to S_y as shown in paragraph 4.4.2 and the value of FS is determined. This computed value of FS is then used in paragraph 4.4.12.2.b.

For $\lambda_c = 0.15$, (Local Buckling)

$$F_{xa} = \min\left[F_{xa1},\ F_{xa2}\right]$$

$$\frac{D_o}{t} = \frac{151.25}{0.625} = 242.0$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{147.0}{\sqrt{75.625(0.625)}} = 21.3818$$

Since $135 < \dfrac{D_o}{t} \le 600$, calculate F_{xa1} as follows with an initial value of $FS = 1.0$.

$$F_{xa1} = \frac{466S_y}{FS\left(331 + \dfrac{D_o}{t}\right)} = \frac{466(33600)}{1.0\left(331 + \dfrac{151.25}{0.625}\right)} = 27325.6545\ psi$$

The value of F_{xa2} is calculated as follows with an initial value of $FS = 1.0$.

$$F_{xa2} = \frac{F_{xe}}{FS}$$

$$F_{xe} = \frac{C_x E_y t}{D_o}$$

Since $\dfrac{D_o}{t} \le 1247$, calculate C_x as follows:

$$C_x = \min\left[\frac{409\bar{c}}{\left(389 + \dfrac{D_o}{t}\right)},\ 0.9\right]$$

Since $M_x \ge 15$, calculate $\bar{c}$ as follows:

$$\bar{c} = 1.0$$

$$C_x = \min\left[\frac{409(1.0)}{389 + \dfrac{151.25}{0.625}},\ 0.9\right] = 0.6482$$

Therefore,

$$F_{xe} = \frac{0.6482(28.3E+06)(0.625)}{151.25} = 75801.9008\ psi$$

$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{75801.9008}{1.0} = 75801.9008\ psi$$

$$F_{xa} = \min[27325.6545,\ 75801.9008] = 27325.6545\ psi$$

With a value of $F_{ic} = F_{xa} = 27325.6545$, in accordance with paragraph 4.4.2, the value of FS is determined as follows.

Since $\{0.55S_y = 0.55(33600) = 18480\} \le \{F_{ic} = 27325.6545\} \le \{S_y = 33600\}$,

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{27325.6545}{33600}\right) = 1.8044$$

Using this computed value of $FS = 1.8044$ in paragraph 4.4.12.2.b, F_{xa} is calculated as follows.

$$F_{xa1} = \frac{466S_y}{FS\left(331+\dfrac{D_o}{t}\right)} = \frac{466(33600)}{1.8044\left(331+\dfrac{151.25}{0.625}\right)} = 15143.9007\ psi$$

$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{75801.9008}{1.8044} = 42009.4773\ psi$$

$$F_{xa} = \min[15143.9007,\ 42009.4773] = 15143.9007\ psi$$

Compare the calculated axial compressive membrane stress, σ_{sm} to the allowable axial compressive membrane stress, F_{xa} per following criteria

$$\{\sigma_{sm} = 2803.9\ psi\} \le \{F_{xa} = 15143.9\ psi\} \qquad True$$

Therefore, local buckling due to axial compressive membrane stress is not a concern.

Table E4.15.2.1 - Design Loads and Load Combinations from VIII-2

Table 4.1.1 – Design Loads

Design Load Parameter	Description
P	Internal or External Specified Design Pressure (see paragraph 4.1.5.2.a)
P_s	Static head from liquid or bulk materials (e.g. catalyst)
D	Dead weight of the vessel, contents, and appurtenances at the location of interest, including the following: • Weight of vessel including internals, supports (e.g. skirts, lugs, saddles, and legs), and appurtenances (e.g. platforms, ladders, etc.) • Weight of vessel contents under operating and test conditions • Refractory linings, insulation • Static reactions from the weight of attached equipment, such as motors, machinery, other vessels, and piping • Transportation Loads (The static forces obtained as equivalent to the dynamic loads experienced during normal operation of a transport vessel – see paragraph 1.2.1.2.b)
L	• Appurtenance Live loading • Effects of fluid flow, steady state or transient • Loads resulting from wave action
E	Earthquake loads (see ASCE 7 for the specific definition of the earthquake load, as applicable)
W	Wind Loads (See 4.1.5.3.b)
S	Snow Loads
F	Loads due to Deflagration

Table 4.1.2 – Design Load Combinations

Design Load Combination (1)	General Primary Membrane Allowable Stress (2)
$P+P_s+D$	S
$P+P_s+D+L$	S
$P+P_s+D+S$	S
$0.9P+P_s+D+0.75L+0.75S$	S
$0.9P+P_s+D+\left(0.6W\, or\, 0.7E\right)$	S
$0.9P+P_S+D+0.75\left(0.6W\, or\, 0.7E\right)+0.75L+0.75S$	S
$0.6D+\left(0.6W\, or\, 0.7E\right)$ (3)	S
P_s+D+F	See Annex 4.D

Notes
1) The parameters used in the Design Load Combination column are defined in Table 4.1.1.
2) S is the allowable stress for the load case combination (see paragraph 4.1.5.3.c)
3) This load combination addresses an overturning condition for foundation design. It does not apply to design of anchorage (if any) to the foundation. Refer to ASCE/SEI 7-10, 2.4.1 Exception 2 for an additional reduction to W that may be applicable.

Table E4.15.2.2 - Design Loads (Net-Section Axial Force and Bending Moment) at the Location of Interest

Design Load Parameter	Description	Magnitude of Pressure, Force and Moment
P	Internal or External Specified Design Pressure (see paragraph 4.1.5.2.a)	$P = 0.0$
P_s	Static head from liquid or bulk materials (e.g. catalyst)	$P_s = 0.0$
D	The dead weight of the vessel including skirt, contents, and appurtenances at the location of interest	$D_F = -363500\ lbs$ $D_M = 0.0\ in-lbs$
L	Appurtenance live loading and effects of fluid flow	$L_F = -85700\ lbs$ $L_M = 90580\ in-lbs$
E	Earthquake loads	$E_F = 0.0\ lbs$ $E_M = 18550000\ in-lbs$
W	Wind Loads	$W_F = 0.0\ lbs$ $W_M = 29110000\ in-lbs$
S	Snow Loads	$S_F = 0.0\ lbs$ $S_M = 0.0\ in-lbs$
F	Loads due to Deflagration	$F_F = 0.0\ lbs$ $F_M = 0.0\ in-lbs$

Based on these loads, the shell is required to be designed for the load case combinations shown in Table E4.15.2.3. Note that this table is given in terms of the load combinations shown in Table 4.1.2 (Table E4.15.2.1 of this example).