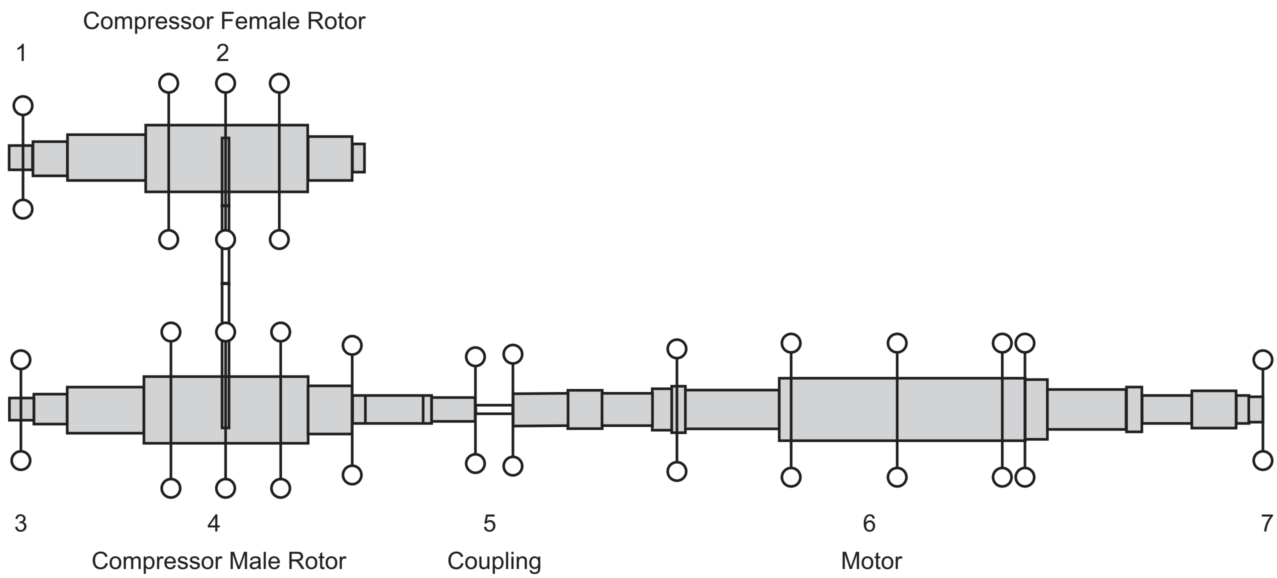

Figure 4-29—Typical System Model of a Flooded Compressor Train

4.2.8 Steam Turbines

A steam turbine rotor is normally modeled as a flexible shaft without considering blade flexibility. Turbine rotor designers typically assume rigid blade-disc systems, whereas blade designers assume a fixed boss. This type of modeling is sufficient for most torsional systems especially when considering the types of turbines used in the petroleum, chemical, and gas industry services where the blade natural frequencies are typically much higher than the torsional excitation sources.

However, such assumptions can be inadequate in those cases where the first tangential eigenfrequency of the independent blade systems is less than or near an excitation frequency such as twice the line frequency of a turbine-generator string, thereby, requiring a coupled blade-disc-rotor vibration analysis [19]. These cases are typically restricted to strings of equipment having turbines with long last stage blades such as large turbine-generator sets as would be found in a central power station. In such trains, the only component typically requiring the coupled model is the low pressure turbine. ISO sites some major incidents due to modes of the coupled shaft and blade system that were resonant with the grid excitation frequencies that occurred in the 1970s [20]. In such cases, the blades may be treated as individual branch elements. Other techniques are also available [21].

4.2.9 References

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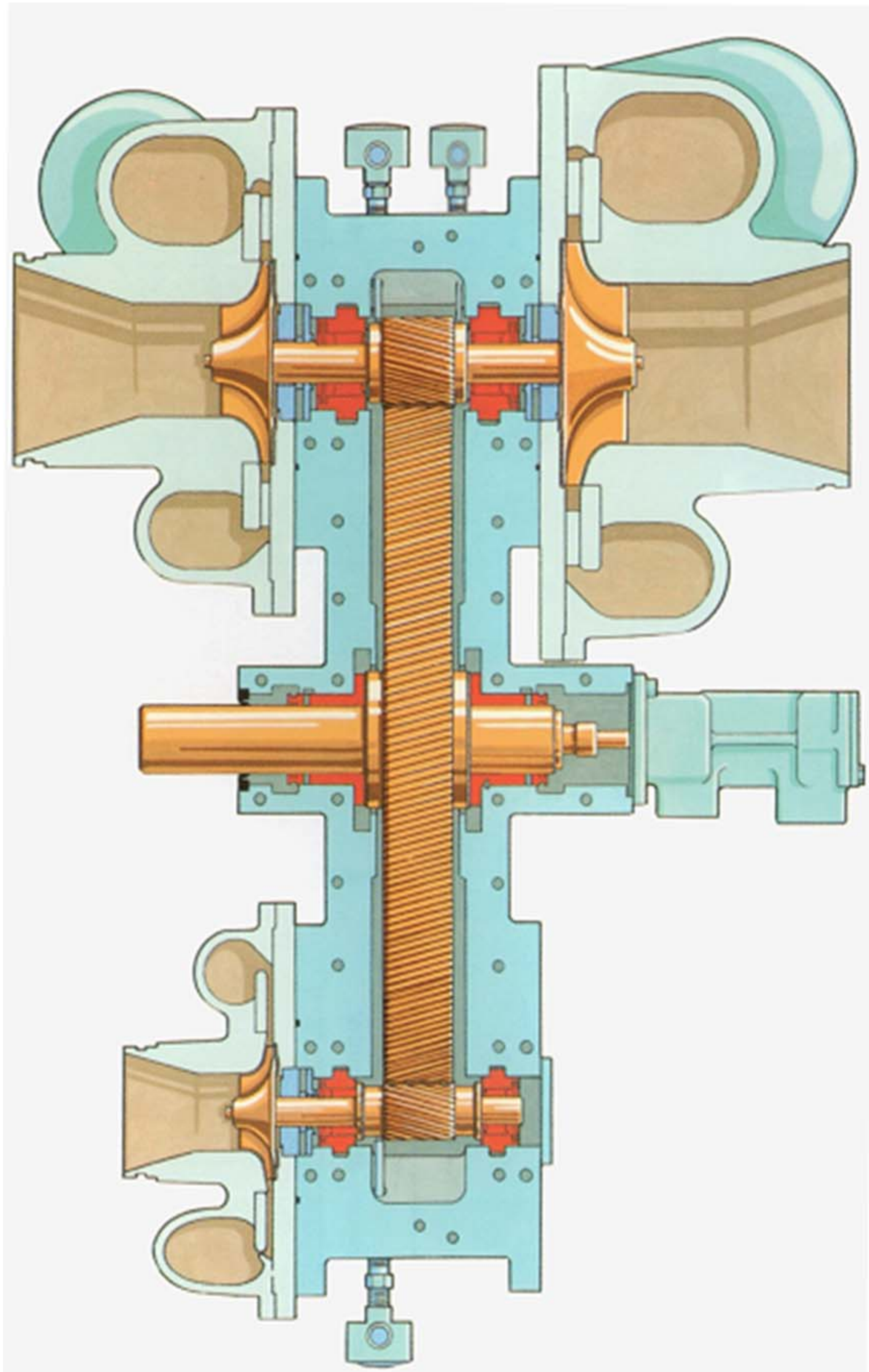

Figure 4-30—API 672 Integrally Geared Compressor

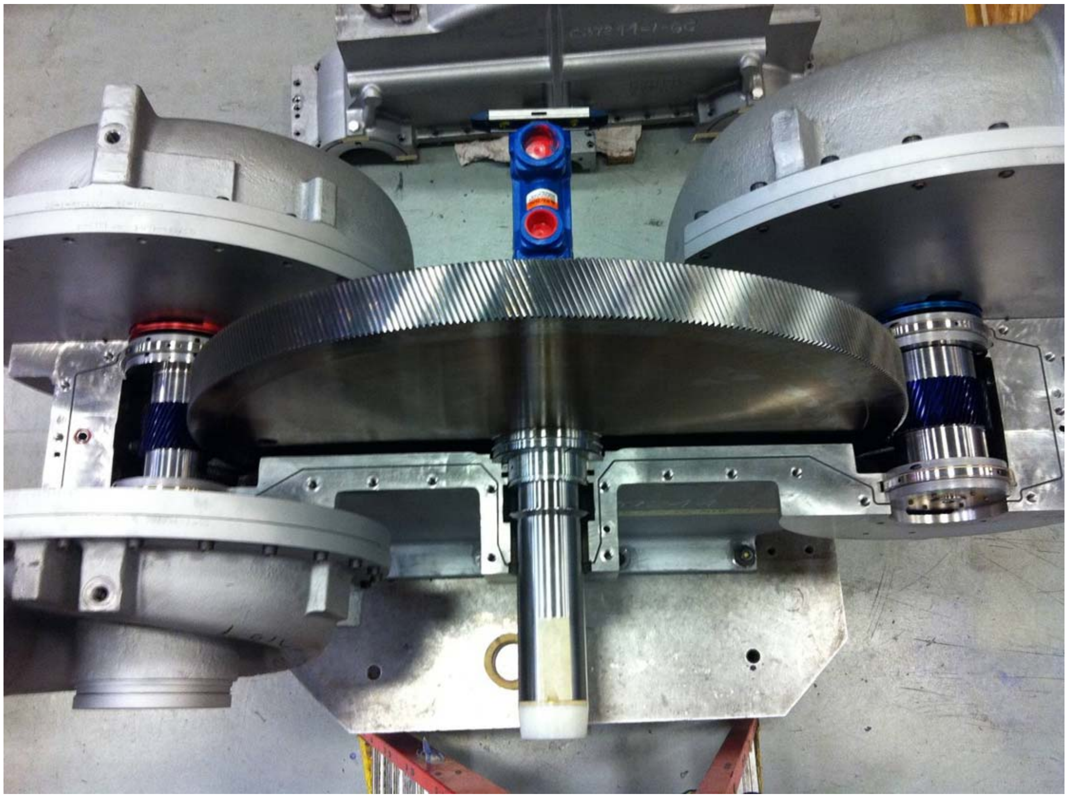

Figure 4-31—API 672 Integrally Geared Compressor

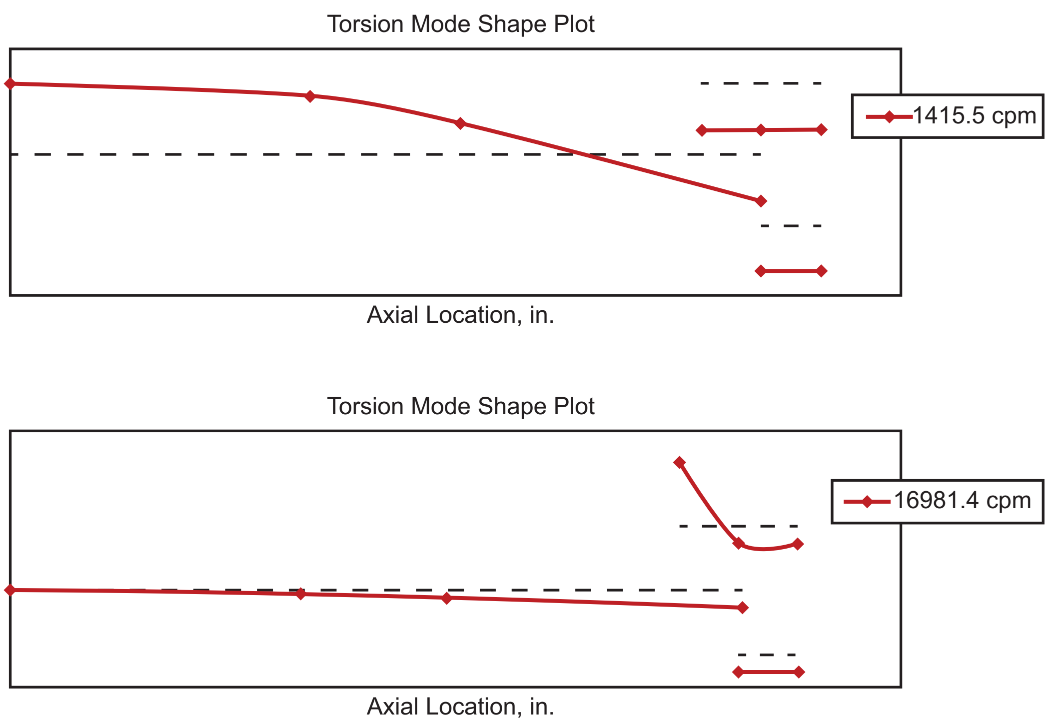

Figure 4-32—Typical Mode Shapes for an Integrally Geared Compressor

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4.3 Reciprocating Machinery

4.3.1 Scope

This section departs from the general format of the entire document pertaining to torsional vibration in that it does not separate the different aspects of modeling a reciprocating machine and excitation from reciprocating machinery into different sections. The task force was of the opinion that the subject of torsional vibrations of reciprocating machinery would be better presented as a separate subject complete onto itself. However even though reciprocating machinery creates torsional excitation and the modeling of reciprocating machinery is unique to itself, there can be other torsional aspects which are presented in the rest of the document such as transient vibration of synchronous motor-driven reciprocating compressors or electric variable frequency drives (VFD) or torsional excitation associated with motor fault conditions that must also be considered in addition to the potential nonuniform torque associated with reciprocating machinery.

4.3.2 Modeling of Reciprocating Machinery Modeling of Reciprocating Machinery

4.3.2.1 General

For more complicated geometries such as crankshafts, the following procedure can be used if the mass-elastic model is not provided by the manufacturer. A crankshaft can be simplified into several main components: the stub shaft that connects to a coupling or flywheel, journals where the main bearings are located, webs, and crankpins. A mass-elastic model of the crankshaft is typically created by lumping the rotating and effective reciprocating inertia at each throw and calculating the equivalent torsional stiffness between throws. Additional mass stations are created for the flywheel, oil pump, etc., as necessary.

Figure 4-33 shows a basic crankshaft throw. A throw consists of two webs and a crankpin. Depending on the type of crankshaft, there may be one or two throws between journals. The crankpin usually drives a connecting rod, crosshead (for compressors) and a piston and piston rod. Engines with power cylinders in a "V" arrangement may have two connecting rods at each crankpin, or use an articulated rod design. Integral engine/compressor units can have two power cylinders articulated off the main connecting rod for the compressor cylinder for a total of three connecting rods per throw.

4.3.2.2 Modeling Crankshaft Torsional Stiffness

As is described by Feese and Hill [1], the determination of the effective torsional stiffness between crankshaft throws is not a simple task. Equations are given in Nestorides [2] and Ker Wilson [3] for calculating the torsional stiffness of a crankshaft. The basic dimensions of the journals, webs, and crankpins are needed, as well as the shear modulus of the shaft material. BICERA also developed curves based on test data for various types of crankshafts. These can be more accurate, but also more complex and are not discussed here.

Carter's Formula [1]

$$K_{\mathrm{t}} = \frac{\pi G}{32\left[\dfrac{L_{\mathrm{j}} + 0.8L_{\mathrm{w}}}{D_{\mathrm{j}}^4 - d_{\mathrm{j}}^4} + \dfrac{0.75L_{\mathrm{c}}}{D_{\mathrm{c}}^4 - d_{\mathrm{c}}^4} + \dfrac{1.5R}{L_{\mathrm{w}}W^3}\right]} \tag{4-11}$$

Ker Wilson's Formula [3]

$$K_{\mathrm{t}} = \frac{\pi G}{32\left[\dfrac{L_{\mathrm{j}} + 0.4D_{\mathrm{j}}}{D_{\mathrm{j}}^4 - d_{\mathrm{j}}^4} + \dfrac{L_{\mathrm{c}} + 0.4D_{\mathrm{c}}}{D_{\mathrm{c}}^4 - d_{\mathrm{c}}^4} + \dfrac{R - 0.2(D_{\mathrm{j}} + D_{\mathrm{c}})}{L_{\mathrm{w}}W^3}\right]} \tag{4-12}$$

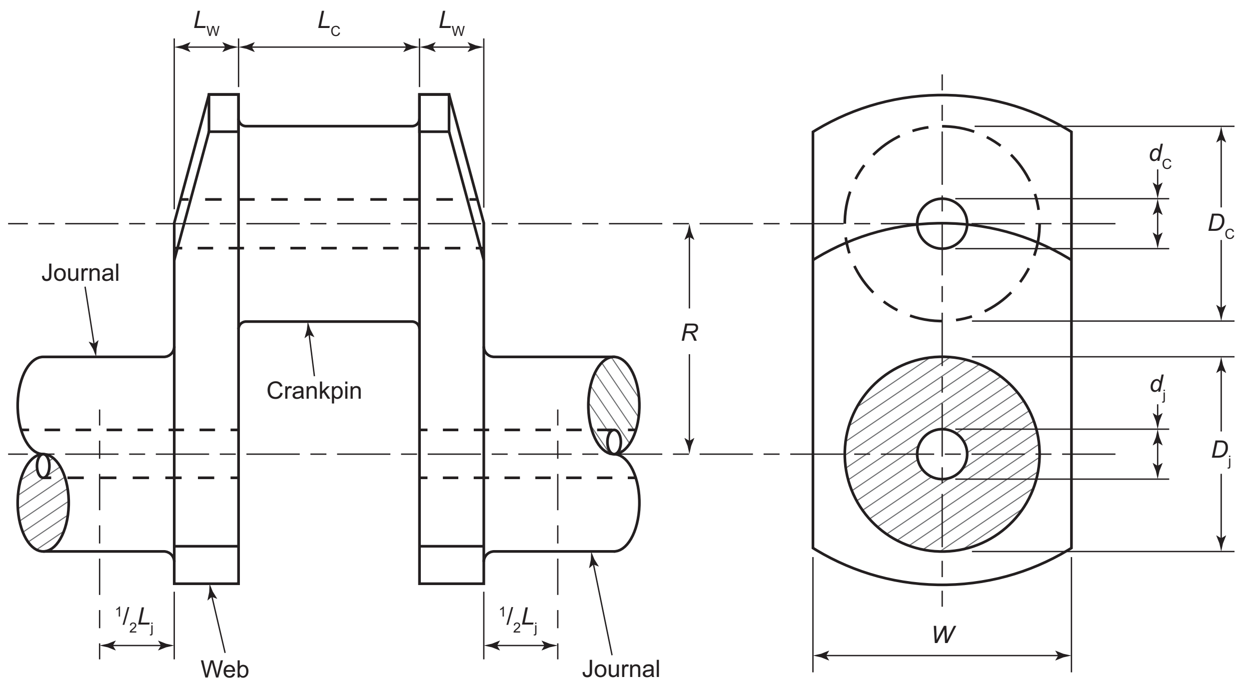


Figure 4-33—Portion of a Typical Crankshaft Throw

where

d_c is the crankpin inside diameter, m (in.);

D_c is the crankpin outside diameter, m (in.);

d_j is the journal inside diameter, m (in.);

D_j is the journal outside diameter, m (in.);

G is the shear modulus, N/m^2 (lbf-in.^{2});

K_t is the torsional stiffness, N-m/rad (lbf-in./rad);

L_c is the crankpin length, m (in.);

L_j is the crankshaft journal length, m (in.);

L_w is the web thickness, m (in.);

R is the throw radius, m (in.);

W is the web width, m (in.).

Carter's formula is applicable to crankshafts with flexible webs and stiff journals and crankpins, while Ker Wilson's formula is better for stiff webs with flexible journals and crankpins. When conducting a torsional analysis, Ker Wilson has suggested using the average of his and Carter's formulas to determine the stiffness between throws. To calculate the torsional stiffness of the stub shaft to the centerline of the first throw, the torsional stiffness of the straight shaft section can be combined in series with twice the torsional stiffness between throws. For coupling hubs or flywheels with an interference fit, the $^{1}/_{3}$ rule should also be applied.

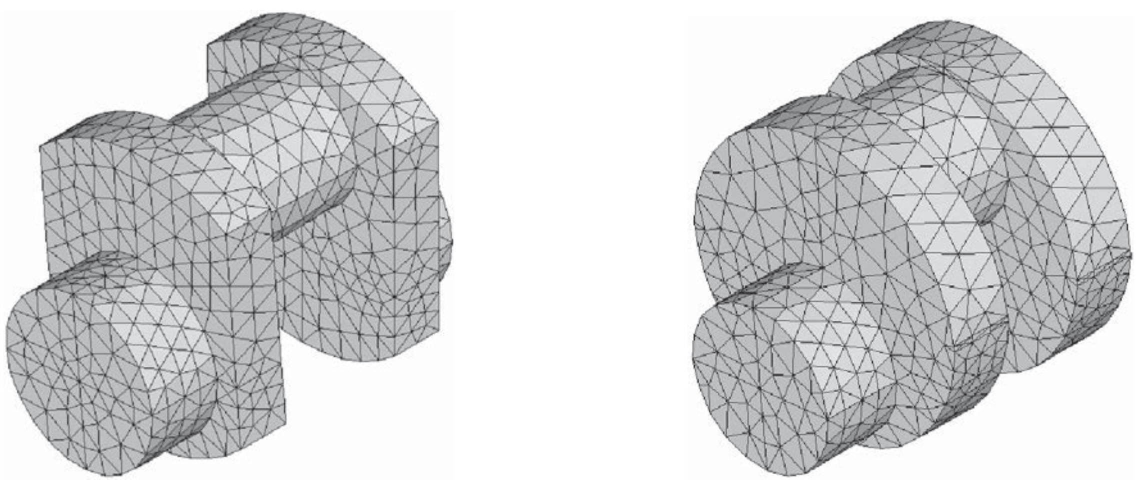
Equation 4-11 and Equation 4-12 were developed before finite element analysis (FEA) was readily available. A finite element program can be used to determine the torsional stiffness for a crankshaft section. The simple models shown in Figure 4-34 were developed from the basic dimensions for two different crankshafts and do not include fillet radii and oil holes. These models are from journal centers, which is the same as the distance between throw centers. One end was rigidly fixed and a moment was uniformly applied across the other end. The calculated torsional stiffness is equal to the moment divided by the angle of twist at the free end. It is interesting to note that for both crankshafts, the calculated torsional stiffness using FEA fell between the values from the Carter and Ker Wilson formulas.

4.3.2.3 Polar Mass Moment of Inertia

The polar mass moment of inertia (commonly referred to as WR^2) at each throw depends on the rotating inertia and the reciprocating mass. The rotating inertia is constant, but the effective inertia of the reciprocating parts actually varies during each crankshaft rotation. This effect is considered to be negligible for most engines except in the case of large slow-speed marine applications [4] when calculating the torsional natural frequencies. As is discussed in Section 4.3.3.2, the reciprocating masses also produce dynamic torque that must be included in the applied torque-effort for forced response calculations.

Carter's Formula = 69.7 x 10^6 N-m/rad (617 x 10^6 lbf-in./rad)
Ker Wilson's Formula = 61.46 x 10^6 N-m/rad (544 x 10^6 lbf-in./rad)
ANSYS Results = 63.72 x 10^6 N-m/rad (564 x 10^6 lbf-in./rad)

Carter's Formula = 154.68 x 10^6 N-m/rad (1369 x 10^6 lbf-in./rad)
Ker Wilson's Formula = 121.35 x 10^6 N-m/rad (1074 x 10^6 lbf-in./rad)
ANSYS Results = 124.96 x 10^6 N-m/rad (1106 x 10^6 lbf-in./rad)



L_j = 0.1461 m (5.75 in.)
L_c = 0.273 m (10.75 in.)
L_w = 0.102 m (4 in.)
R = 0.216 m (8.5 in.)
D_j = 0.33 m (13 in.)
D_c = 0.254 m (10 in.)
W = 0.42 m (16.5 in.)
G = 79.29 x 10^6 N/m^2 (11.5 x 10^6 psi in.)

L_j = 0.168 m (6.625 in.)
L_c = 0.178 m (7 in.)
L_w = 0.132 m (5.1875 in.)
R = 0.267 m (10.5 in.)
D_j = 0.33 m (13 in.)
D_c = 0.305 m (12 in.)
W = 0.635 m (25 in.)
G = 79.29 x 10^6 N/m^2 (11.5 x 10^6 psi in.)

Figure 4-34—Finite Element Models Used to Calculate Torsional Stiffness of Crankshaft Sections

The equivalent inertia, I_{eqv}, at each throw can be approximated by adding the rotating inertia of the crankshaft section, I_{rot}, to half of the reciprocating mass, M_{recip}, times the throw radius, R, squared.

$$I_{eqv} \approx I_{rot} + 0.5 \times M_{recip} R^2 \tag{4-13}$$

where

I_{eqv} is the equivalent inertia, kg-m^2 (lbm-in.^{2});

I_{rot} is the rotational inertia of crankshaft throw, kg-m^2 (lbm-in.^{2});

M_{recip} is the reciprocating mass, kg (lbm);

R is the throw radius, m (in.).

The rotational inertia of the journal and crankpin can be calculated using the equation for a cylinder. Since the crankpin rotates at the throw radius and not about its center, the parallel axis theorem must also be used. The inertia of the webs can be estimated with an equation for a rectangular prism. Any rotating counter-weights that may be bolted to a web should also be included. There are solid model 3D CAD packages that can perform this calculation.

Since part of the connecting rod is rotating and part is reciprocating, it is common practice to divide the rod's total mass into two portions—a rotating portion and a reciprocating portion. The rotating portion can be calculated using Equation 4-14. The connecting rod is generally heavier at the crankpin end and lighter at the reciprocating end. If the weight distribution of the connecting rod is unknown, a typical assumption is to use two-thirds of the weight as rotating and one-third as reciprocating. The rotating mass of the connecting rod is multiplied by the throw radius squared and added to the crankshaft rotating inertia to obtain the total rotational inertia, I_{rot}. The total reciprocating mass includes the small end of the connecting rod, cross-head (for compressors), nut, piston, and piston rod. This total mass needs to be used in Equation 4-13 when calculating the throw inertia.

$$M_{rot} = [(L_{rod} - L_{cg})/L_{rod}] \times M_{rod} \tag{4-14}$$

where

M_{rot} is the mass of rotating portion of connecting rod, kg (lbm);

L_{rod} is the total length of connecting rod, m (in.);

L_{cg} is the distance from crank pin center to rod CG, m (in.);

M_{rod} is the total mass of connecting rod, kg (lbm).

4.3.3 Torsional Excitations Generated by Reciprocating Machinery

4.3.3.1 General

Reciprocating compressors and engines produce unsteady torque. This torque variation can be much higher than in rotating equipment and flywheels are often used to smooth the torque. The frequencies of the torque excitation should be considered to avoid coincidence with torsional natural frequencies, which could potentially cause problems.

4.3.3.2 Torque Variation Due to Inertial and Gas Forces

From a torsional standpoint, there are two types of forces that cause torque variation at each throw: inertial and gas load. The total force times the distance between the crankshaft centerline and throw centerline is equal to the moment imposed on the crankshaft. At top dead center (TDC) and bottom dead center (BDC), the throw is inline with the connecting rod and piston so that no moment can be imposed on the crankshaft. At 90 degrees from BDC and TDC, the moment arm is at the maximum length (full crank radius).

The rotating inertia of the crankshaft must be considered in the mass-elastic model, but does not cause any torque variation. The inertial forces are caused by the reciprocating mass of the connecting rod, cross-head and piston, which are dependent on the crank angular position and cannot be eliminated by balancing. The unbalance forces have components which vary once per revolution (primary forces), twice per revolution (secondary forces), and three times per revolution (tertiary forces). These inertia forces will vary with the speed squared. Per Den Hartog [5], Equation 4-15 can be used to calculate the 1X, 2X, and 3X components of the inertia excitations. It should be noted that positive torques represent torques being applied to the crankshaft while negative values mean torque is being taken from the crankshaft. These signs are important and must be considered when combining inertia excitations with gas load excitations.

$$\tau_{\text{inertia}} = \frac{1}{2} \times m_{\text{recip}} \times \omega^2 \times R^2 \times \left[\frac{R}{2L}\sin(\omega t) - \sin(2\omega t) - \frac{3R}{2L}\sin(3\omega t)\right] \tag{4-15}$$

where

τ_{inertia}	is the inertia torque acting on crankshaft, N-m (ft-lbf);
ωt	is the crank angle (from top dead center), radians;
m_{recip}	is the total mass of reciprocating parts, kg (slugs);
R	is the crank radius, m (ft);
L	is the connecting rod length, m (ft);
ω	is the rotational speed, rad/sec.

Gas load excitations arise because as a reciprocating compressor or engine goes through its performance cycle, the pressure within its cylinders varies periodically, as a function of crank angle. The gas force is equal to the differential pressure across the piston times the cross-sectional area of the bore. The stroke, or travel of the piston, is equal to twice the crank throw radius. The swept volume for each cylinder is the bore area times the stroke. In order to determine the gas load excitations, the pressure versus crank angle must be determined for each cylinder over 360 degrees for compressor or two-stroke engine and 720 degrees for a four-stroke engine. The torque can then be determined versus crank angle by simply multiplying the pressure by the bore area and the crank radius. Any distortion in the pressure waveform will affect the dynamic torque and torsional response. A third type of curve that is often seen is called tangential effort (or tangential pressure), which is the torque normalized for unity crank radius and piston area.

Once the inertia and gas forces have been determined, they must be correctly added together for each cylinder. The torque at each throw must then be properly phased for the entire machine. A Fourier analysis can then be performed on these curves to represent the complex wave as a series of sinusoidal curves at various harmonics. At each harmonic, the amplitude and phase can be calculated, or the values can be presented as coefficients of sine and cosine functions. Compressors and two-stroke engines will have integer harmonics of running speed while four-stroke engines produce both integer and half orders. Depending on the cylinder phasing, certain orders may cancel out while others become dominant when examining the overall torque output from the machine.

Compressor and engine manufacturers will often provide this information in various forms with the performance calculations. To use their data in a torsional analysis, it is very important to understand the sign convention and if the values are only for gas forces or if the effect of reciprocating mass has also been included. Computer programs are used to calculate the torsional excitation for compressors and engines.

4.3.3.3 Compressors

Ideal pressure cards are often used for analysis since they can be computed from the compressor and gas properties. However, ideal cards do not include valve/manifold losses and gas pulsation, which can affect the resulting torque harmonics. Situations where the harmonic content is changed and the torsional excitation by the compressor could be increased are: valve failure, gas pulsation due to an acoustic resonance, and various load steps.

Single-acting (SA) compressors use only one side of the piston while double-acting (DA) cylinders use both the crank and head ends. A compressor valve failure can be analyzed by unloading one end of a cylinder. All load steps must be considered in the analysis, such as unloaders, pockets, etc., as these load steps can significantly affect the harmonic content and influence the torsional responses. The maximum horsepower case will not necessarily correspond to the maximum torsional excitation at all harmonics. The full range of operating conditions (pressures, flows, gas mole weights, etc.) should also be considered.

The torsional behavior of reciprocating compressors has been found to be highly sensitive to normal tolerance variations. That is, a compressor that can be shown to be acceptable via analysis of the nominal system could fail if certain tolerance conditions were to occur. For that reason, it is recommended that tolerance variations be accounted for when analyzing a reciprocating compressor. References [6–8] give more detail on how to implement this.

4.3.3.4 Engines

For a two-stroke engine, intake, compression, expansion, and exhaust occur during one revolution of the crankshaft. However, with four-stroke engines, these cycles occur over two revolutions, which causes half-order excitations. Some engines have a choice of firing orders, which can change the strong harmonics. In critical systems, the best firing order could be chosen to reduce the torsional response.

Poorly maintained engines will tend to operate at nonideal conditions that can cause high torsional vibration such as: engine misfire, pressure imbalance, ignition problems, and leaks. Misfire is common when the fuel is inconsistent, such as biogas from waste treatment or landfills.

A misfire condition should be analyzed by assuming one cylinder does not fire. Since the response can vary substantially depending on which cylinder misfires, the worst case should be assumed. In many practical systems, the misfire case is the sternest test of the design. Thus, the misfire case should be evaluated in all torsional analyses involving engines.

4.3.4 Steady-state Response Analysis

The steady-state response analysis for reciprocating trains is fairly similar to that for purely rotary trains, which is described in Section 4.4. However, there are several significant differences.

First, in a rotary train, there are common situations where response analysis is not even required. Specifically, if there are no steady-state interference points in or near the operating speed range, response analysis can usually be omitted. On the other hand, response analysis almost always needs to be performed for a reciprocating train.

Second, in a rotary train, response analysis is only performed at a few specific speeds—the resonant speeds. Conversely, since there are so many active excitations in a reciprocating train, limiting the analysis to a few governing speeds is often impossible. Thus, most reciprocating trains are analyzed using a “speed sweep” response analysis, as is shown in Figure 4-35.

Third, in a rotary train, there is only one response of interest—that due to the resonant excitation. On the other hand, in a reciprocating train, all excitations need to be included in the response analysis. Additionally, as is shown in Figure 4-35, the response obtained by summing the responses due to all the individual excitations, with phasing accounted for (which is shown by the solid line in the figure), is usually significantly larger than that due to any single excitation

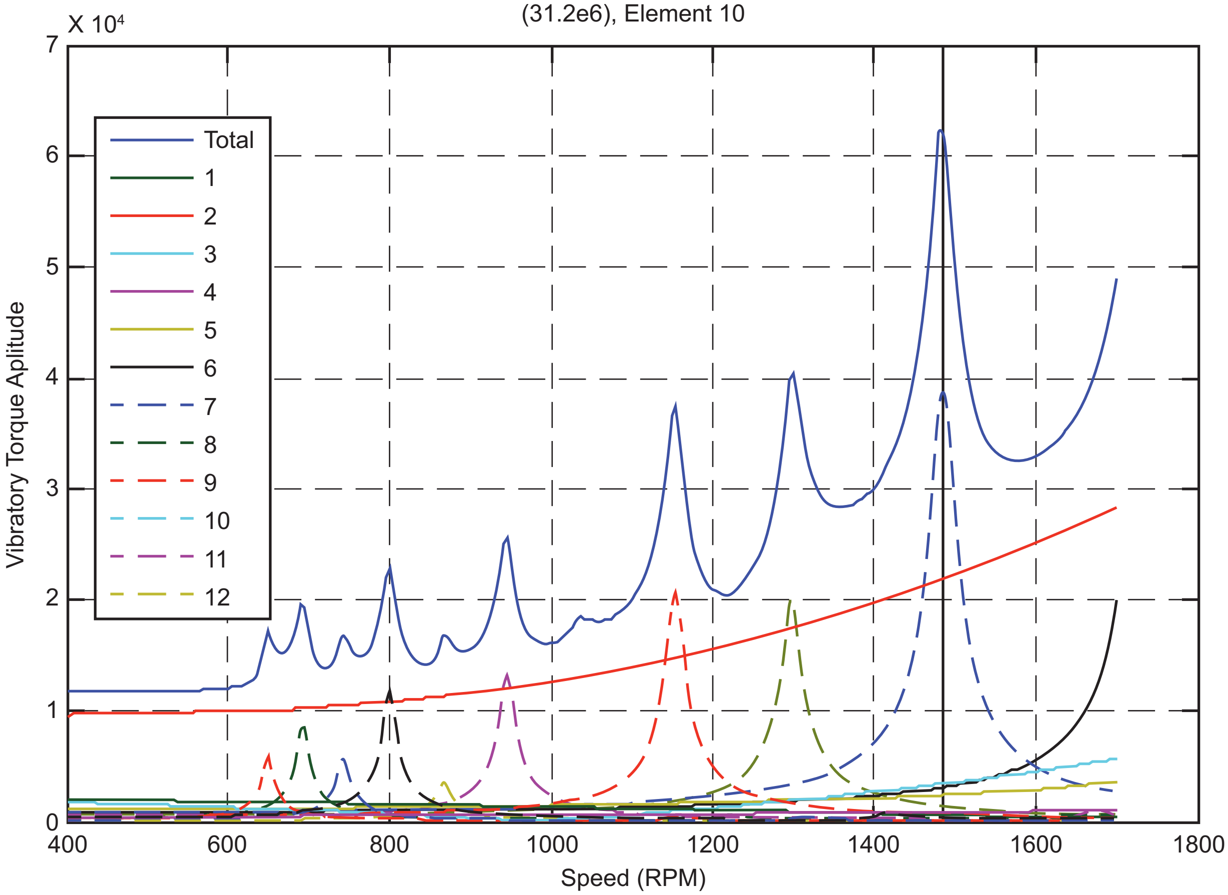


Figure 4-35—Typical Reciprocating Train Response Plot

(represented by the dashed lines), even one at resonance. Thus, the overall summed response is the one that should be compared to the acceptability criteria.

Finally, some of the acceptability criteria are different from those normally used for purely rotary trains. In general, the items that need to be checked for a reciprocating train are as follows:

1) All shafts must have adequate fatigue life. This is similar to rotary systems (and, thus, can be evaluated using the procedure given in Section 4.6) except stresses in the crankshaft must also be looked at. This is discussed further in Section 4.3.5.

2) Torques in couplings must be below manufacturers' limits. This is no different from rotary systems.

3) Motors having spider construction must be checked for their ability to handle the imposed cyclic torques. In reciprocating compressor trains, spider rotors in motors have often been observed to be the weak link. Thus, the spider's ability to handle the calculated cyclic torques needs to be verified with the motor supplier.

4) Cyclic torques at gear meshes must be below applicable limits (usually specified as a percentage of the torque transmitted through the mesh).

5) The displacements and accelerations at the free ends of both reciprocating engines and compressors must be below applicable limits. There are often auxiliary components (which aren't included in the torsional model) attached to the free ends of engines or compressors which can break if there is too much motion at the free end.