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Improved Finite Element Model for Calculating Stresses in Bevel and Hypoid Gear Teeth

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TECHNICAL PAPER

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Abstract

Recent advances in the speed and memory capacity of current computers combined with H–adaptive finite element theory permit the development of more realistic finite element gear strength models. The speed and memory improvements allow for increased numbers of elements and degrees of freedom and the addition of more detailed base regions underneath the gear teeth of the finite element model. The H–adaptive theory increases the accuracy of the finite element model by optimizing the size and shape of individual elements within the model. This paper presents results comparing the predicted fillet strain output of a three–dimensional gear tooth model with recently obtained experimental strain gage data. Comparisons are made for both spiral bevel and hypoid gears. Preliminary results show excellent agreement between theory and experiment with peak strain amplitudes agreeing to within ten percent or less. The inclusion of more accurate base regions underneath the gear teeth correctly predicts the range of strain from tensile to compressive values as the gear teeth roll through mesh.

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Introduction

The bending fatigue life of bevel and hypoid gears is directly related to the magnitude of the tensile bending stress in the fillet regions of the gear teeth. It is possible, therefore, to predict the bending fatigue life of a gear design prior to manufacture if the tensile bending stresse can be determined through theoretical stress analysis. As early as 1893 Wilfred Lewis [1] applied cantilever beam theory to two-dimensional models of gear teeth in order to estimate the fillet bending stresses. Further improvements were made to the Lewis formulation by Merrit [2], Baud and Timoshenko [3] and Dolan and Broghamer [4] with the addition of factors accounting for compressive load effects, the influence of tooth proportions and stress concentrations due to fillets. In more recent times, both Coleman [5] and AGMA [6] have extended the two-dimensional stress analysis concept to three-dimensional bevel and hypoid gear teeth by including additional factors to account for the influence of tooth proportions, contact pattern position and load sharing between teeth. Wilcox [7] applied finite element analysis to improve the range and accuracy of the basic two-dimensional stress model.

Each of the previously mentioned stress calculation methods have in common a two-dimensional stress model (or kernel) that is extended to three dimensions by use of multiplying factors. The accuracy of these stress calculation methods is dependent on the accuracy of the two-dimensional kernel and the assumptions made in deriving the various multiplying factors. The overall accuracy is often compromised because it is not possible to account for all possible interactions between variables when using multiplying factors. There is clearly a continuing need for a more accurate and consistent method of calculating gear tooth bending stresses.

In the late nineteen seventies the three-dimensional finite element method of stress analysis began to be used for calculation of stresses in complex three-dimensional structures. Wilcox and Auble [8] developed a three-dimensional stress calculation model based on the finite element method. Although primitive by today's standards this early finite element model was based on accepted and standardized finite element concepts and equally important incorporated no additional multiplying factors.

In 1982 Wilcox [9] modified the three-dimensional finite element program to incorporate the flexibility matrix method. The flexibility matrix method introduced by Conry and Sierig [10] divides the finite element approach into two steps. First, the finite element models for both gear and pinion teeth are solved for a series of point loads distributed over the surfaces of the gear teeth in order to characterize the deflection and stress behavior of the teeth. In the second step, the point load solution data are combined according to a set of linear equations (linear superposition) to calculate the load distributions along each line of contact as the gear and pinion roll through mesh. The flexibility matrix method is the forerunner of the modern "gap" element widely used in today's finite element programs. The combination of the finite element method and the flexibility matrix method resulted in an accurate and efficient method for calculating bending stresses in the fillets of gear teeth.

More recently Vijayakar, Busby and Houser [11] applied "quasi-prismatic" elements incorporating high order Chebyshev polynomials in the direction of the gear tooth face width while using conventional isoparametric displacement polynomials in the two-dimensional cross section of the teeth. This procedure allows for the use of higher order displacement variations in the face width direction of the tooth while maintaining reasonable computer solving times.

During the remainder of the nineteen eighties the accuracy of the combined finite element-flexibility matrix program developed by Wilcox was tested with mixed results. Although the program was able to predict stresses with reasonable accuracy about eighty percent of the time, there were glaring exceptions. Theoretically predicted contact patterns were generally found to be shorter than those observed in experimental loaded tooth contact tests and the calculated bending stresses could on occasion be up to fifty percent more (or less) than those measured using strain gauges in the fillets. Eventually it was determined that the finite element models were too simplistic and suffered from low densities of elements in the critical stress regions and by not including the stress raising effects of loads on the neighboring gear teeth.

Based on a new understanding of some of the deficiencies of the old finite element tooth model and noting the success of using higher order polynomials to model the displacement field an effort was undertaken to develop an improved three-dimensional finite element model. The remainder of this paper outlines the development of a greatly improved finite element gear stress model that is expanded in size and incorporates higher order polynomials to model the displacement field. The accuracy of the finite element based stress program is tested against a bevel and hypoid gear set and the results of the comparison are presented.