The theory is then used to calculate the pitting or bending fatigue lives for gears that are subjected to variable loads. Accordingly, failure could occur when:

$$\frac{n_1}{N_{f1}} + \frac{n_2}{N_{f2}} + \dots + \frac{n_i}{N_{fi}} = 1$$
B.4

where

 $\frac{n_{i}}{N_{fi}}$  is the damage ratio at the *i*-th stress.



Cumulative number of applied cycles

# Figure B.1 – Load spectrum

If the fraction of cycles at each stress is known rather than the actual number of cycles, the cycles are given by:

$$n_{\rm i} = \alpha_{\rm i} N \tag{B.5}$$

where

 $\alpha_i$  is the cycle ratio (fraction of cycles at the *i*-th stress);

N is the resultant fatigue life (total cycles).

Miner's Rule [5] may then be rewritten as:

$$\frac{\alpha_1 N}{N_{f1}} + \frac{\alpha_2 N}{N_{f2}} + \ldots + \frac{\alpha_i N}{N_{fi}} = 1$$
(B.6)

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which may be solved for the resultant life:

$$N = \frac{1}{\frac{\alpha_{1}}{N_{f1}} + \frac{\alpha_{2}}{N_{f2}} + \dots + \frac{\alpha_{i}}{N_{fi}}}$$
(B.7)

The cycle ratio may be obtained from the load spectrum by:

$$\alpha_i = \frac{n_i}{\Sigma n_i} \tag{B.8}$$

where

- $n_i$  is the number of cycles at the *i*-th load in the load spectrum;
- $\Sigma n_i$  is the total number of cycles in the load spectrum. The number of cycles at each load is calculated from:

 $n_{\rm i} = 60 w_{\rm i} t_{\rm i} \tag{B.9}$ 

where

 $w_i$  is the speed at the *i*-th load (rpm);

 $t_i$  is the time at the *i*-th load (hour).

The equivalent (baseline) speed,  $w_b$ , is given by:

$$w_{\rm b} = \frac{1}{\frac{\alpha_1}{w_1} + \frac{\alpha_2}{w_2} + \ldots + \frac{\alpha_i}{w_i}}$$
(B.10)

The resultant life in hours is:

$$L = \frac{N}{60 \times w_{\rm b}} \tag{B.11}$$

See Table B.2 for the example problem calculations.

The slope exponent can be a very small number. As such, small differences in calculated stress levels will produce very great differences in calculated life. As the slope exponent becomes smaller, load segments of the duty cycle representing the heaviest loads have the greatest detrimental effect on gear life.

S-N curves must be established with utmost care so that the effect of the "shape" of the duty cycle is properly considered.

#### B.4 Failure mode

Because of the differences in the S-N curves for pitting and bending, the effective load for pitting and effective load for bending will be different. It is difficult to predict the mode of failure without a complete analysis of the duty cycle. If the shape or magnitude of the duty cycle changes, the mode of failure may also change.

EXAMPLE PROBLEM: Carburized grade 2 gears

where

$$s_{\rm ac} = 225\ 000\ {\rm lb/in^2}$$
  
 $K_{\rm T} = C_{\rm R} = 1.0$   
 $C_{\rm L} = \frac{s_{\rm c}}{s_{\rm ac}}$   
 $N_{\rm fi} = \left(\frac{3.4822}{C_{\rm L}}\right)^{10.0602}$ 

			Cvcles	Cycle ratio
Load	Speed, w <sub>i</sub>	Time, <i>t</i> <sub>i</sub>	$n_i = 60 w_i t_i$	$\alpha_i = \frac{n_i}{n_i}$
no.	(rpm)	(hour)	(cycles)	$\Sigma n_{\rm i}$
1	65	1000	3.90 × 10 <sup>6</sup>	0.0556
2	85	2000	1.02 × 10 <sup>7</sup>	0.1453
3	125	3000	2.25 × 10 <sup>7</sup>	0.3205
4	140	4000	<u>3.36 × 10<sup>7</sup></u>	<u>0.4786</u>
			$\Sigma n_{\rm i} = 7.02 \times 10^7$	1.0000
			Life factor	
Load	Contact stress, $s_c$		$C_{\rm L} = \frac{S_{\rm C}}{1}$	Cycles to failure
no.	(lb/in <sup>2</sup> )		<sup>S</sup> ac	N <sub>fi</sub> (cycles)
1	236 250		1.0500	4.46 × 10 <sup>8</sup>
2	224 120		0.9961	1.07 × 10 <sup>9</sup>
3	211 320		0.9392	2.84 × 10 <sup>9</sup>
4	197 660		0.8785	8.62 × 10 <sup>9</sup>
Resultant life in cycles	s per Eq B.7	4		
N =1	=	1	= 2.3	3×10 <sup>9</sup> cycles
$\frac{\alpha_1}{\alpha_1} + \frac{\alpha_2}{\alpha_2} + \frac{\alpha_2}{\alpha_2}$	$+\frac{\alpha_{i}}{\alpha_{i}}$ 0.0556	$+ \frac{0.1453}{2} + \frac{0.320}{2}$	$\frac{0.4786}{2}$ + $\frac{0.4786}{2}$	
$N_{f1}$ $N_{f2}$	N <sub>fi</sub> 4.46×10 <sup>8</sup>	$1.07 \times 10^9$ 2.84 × 1	10 <sup>9</sup> 8.62×10 <sup>9</sup>	
Equivalent speed per Eq B.10				
1 1 4 4 7				
$w_{b} = \frac{1}{\alpha_{1} + \alpha_{2} + \alpha_{i}} = \frac{1}{0.0556 + 0.1453 + 0.3205 + 0.4786} = 1171 \text{pm}$				
$\frac{1}{w_1} + \frac{1}{w_2} + \dots + \frac{1}{w_i} = \frac{1}{65} + \frac{1}{85} + \frac{1}{125} + \frac{1}{140}$				
Resultant life in hours per Eq B.11				
$L = \frac{N}{60 \times w_{\rm b}} = \frac{2.33 \times 10^9}{60 \times 117} = 331909 \text{ hours}$				

Table B.2 – Calculation table

# Annex C

# (informative) Geometry factors, *I* and *J*

[The foreword, footnotes and annexes, if any, are provided for informational purposes only and should not be construed as a part of ANSI/AGMA 2003-D19, *Rating the Pitting Resistance and Bending Strength of Generated Straight Bevel, Zerol Bevel and Spiral Bevel Gear Teeth.*]

# C.1 Geometry factors, *I* and *J*

This annex is presented using conventional AGMA symbols and units. See Annex C(M) for the same information using ISO symbols and units.

The geometry factors, I and J, evaluate the effects of gear geometry on the gear tooth stresses for pitting resistance and bending strength respectively.

Symbols in this annex are listed in Table C.1. Other symbols are listed in Table 1.

AGMA			
symbol	Description	Units	First used
а	Positive integer		Eq C.41
а <sub>G</sub> , а <sub>Р</sub>	Mean addendums of gear and pinion, respectively	in	Eq C.6, C.5
a <sub>oG</sub> , a <sub>oP</sub>	Outer addendums of gear and pinion, respectively	in	Eq C.6, C.5
b	Positive integer		Eq C.41
b <sub>G</sub> , b <sub>P</sub>	Mean dedendums of gear and pinion, respectively	in	Eq C.46, C.45
$b_{\rm oG}$ , $b_{\rm oP}$	Outer dedendums of gear and pinion, respectively	in	Eq C.46, C.45
F <sub>G</sub> , F <sub>P</sub>	Actual face widths of gear and pinion, respectively	in	Eq C.44, C.43
F <sub>K</sub>	Projected length of instantaneous line of contact in lengthwise direction of the tooth	in	Eq C.106
$f_{I}, f_{J}$	Assumed locations of critical point on tooth for pitting resistance and bending strength, respectively	in	Eq C.32, C.47
Н	Empirical constant used in stress correction formula		Eq C.96
$h_{\rm NG},  h_{\rm NP}$	Load heights from critical section for gear and pinion, respectively	in	Eq C.92, C.79
$J_{G}$ , $J_{P}$	Geometry factors for bending strength for gear and pinion, respectively		Eq C.44, C.43
$K_{\rm fG},~K_{\rm fP}$	Stress concentration and stress correction factors for gear and pinion, respectively		Eq C.97, C.96
Kz	Intermediate variable		Eq C.25
k	Summation limits in summation calculations		Eq C.41
<i>k</i> ′	Factor used in geometry factor calculations		Eq C.7
L	Empirical exponent used in stress correction formula		Eq C.96
M	Empirical exponent used in stress correction formula		Eq C.96
m <sub>F</sub>	Face contact ratio		Eq C.26
mo	Modified contact ratio		Eq C.27
m <sub>p</sub>	Transverse contact ratio		Eq C.24
р	Outer transverse circular pitch	in	Eq C.9
$p_{N}$	Mean normal base pitch	in	Eq C.10
$p_{\sf n}$	Mean normal circular pitch	in	Eq C.11
$p_2$	Auxiliary pitch	in	Eq C.12
$p_{\rm 3G}, p_{\rm 3P}$	Locations of points of load application on path of action for gear and pinion, respectively	in	Eq C.51
$R_{\rm bN}$ , $r_{\rm bN}$	Mean normal base radii for gear and pinion, respectively	in	Eq C.18, C.17
			(continued)

Table C.1 – Symbols used in geometry factor equations

AGMA			
symbol	Description	Units	First Used
$R_{\rm N}$ , $r_{\rm N}$	Mean normal pitch radii for gear and pinion, respectively	in	Eq C.16, C.15
$R_{\rm oN}$ , $r_{\rm oN}$	Mean normal outside radii for gear and pinion, respectively	in	Eq C.20, C.19
$r_{\rm fG},r_{\rm fP}$	Tooth fillet radii in mean section at the tooth root circle for gear and pinion, respectively	in	Eq C.69, C.68
$r_{\rm TG},r_{\rm TP}$	Tool or cutter tip edge radii used to produce the gear and pinion, respectively	in	Eq C.69, C.68
t <sub>NG</sub> , t <sub>NP</sub>	One-half tooth thicknesses at critical section for gear and pinion, respectively	in	Eq C.91, C.78
$t_{\rm G}, t_{\rm P}$	Mean normal circular tooth thicknesses for gear and pinion, respectively	in	Eq C.61, C.60
$X_{NG}, X_{NP}$	Tooth strength factors for gear and pinion, respectively	in	Eq C.94, C.81
$X_{\theta G}, X_{\theta P}$	Assumed distances in locating weakest section for gear and pinion, respectively	in	Eq C.85, C.72
<i>x</i> ″ <sub>o</sub>	Distance from mean section to center of pressure measured in the lengthwise direction along the face width	in	Eq C.54
x <sub>oG</sub> , x <sub>oP</sub>	Distances from centerline of crown gear (tool) space to center of tool tip edge radius measured in mean normal section for gear and pinion, respectively	in	Eq C.84, C.71
$x''_{oc}, x''_{op}$	Factors used for calculation for gear and pinion, respectively	in	Eq C.67, C.66
$Y_{\rm G}, Y_{\rm P}$	Tooth form factors excluding stress concentration factors for gear and pinion, respectively		Eq C.95 C.82
$Y_{\rm KG}, \ Y_{\rm KP}$	Tooth form factors for gear and pinion, respectively	in	Eq C.102, C.101
У <sub>2G</sub> , У <sub>2Р</sub>	Distances from center of tool tip edge radius to crown gear pitch surface measured in a direction perpendicular to pitch surface for gear and pinion, respectively	in	Eq C.83, C.70
$Z_{G}', Z_{P}'$	Lengths of addendum action in mean normal section for gear and pinion, respectively	in	Eq C.22, C.21
$Z_{N}$	Length of action in mean normal section	in	Eq C.23
<sup>z</sup> 1G, <sup>z</sup> 1P	Intermediate variables for calculating tooth strength factor for gear and pinion, respectively	in	Eq C.88, C.75
<sup>z</sup> <sub>2G</sub> , <sup>z</sup> <sub>2P</sub>	Intermediate variables for calculating tooth strength factor for gear and pinion, respectively	in	Eq C.89, C.76
Z <sub>o</sub>	Distance along path of action in mean normal section from pitch line to point of maximum contact stress	in	Eq C.35
α <sub>G</sub> , α <sub>P</sub>	Addendum angles of gear and pinion, respectively		Eq C.4, C.3
Γ, γ	Pitch angles (reference cone angles) of gear and pinion, respectively		Eq C.4, C.3
Γ <sub>ο</sub> , γ <sub>ο</sub>	Face angles (tip angles) of gear and pinion, respectively		Eq C.4, C.3
$\Delta F_{\mu C}, \Delta F_{\mu D}$	Heel increments of face width for gear and pinion, respectively	in	Eq C.115,
$\Delta F'_{\rm HG}, \Delta F'_{\rm HP}$	('effective)		C.110, C.113, C.108
$\Delta F_{TG}, \Delta F_{TP}$ $\Delta F'_{TG}, \Delta F'_{TP}$	Toe increments of face width for gear and pinion, respectively ('effective)	in	Eq C.114, C.109, C.112, C.107
$\Delta R_{\rm N}, \Delta r_{\rm N}$	Distances from pitch circle to point of load application in mean normal section for gear and pinion, respectively	in	Eq C.65, C.64
$δ_{\sf G}$ , $δ_{\sf P}$	Dedendum angles of gear and pinion, respectively		Eq C.46, C.45
η	Length of action within the contact ellipse	in	Eq C.29

Table	C.1	(continued)

(continued)

AGMA			
symbol	Description	Units	First Used
$\theta_{\text{hG}},\theta_{\text{hP}}$	One half of angles subtended by normal circular tooth thickness		Eq C.61, C.60
$\theta_{G},  \theta_{P}$	Assumed angles in locating weakest section for gear and pinion, respectively respectively		Eq C.86, C.73
$\rho_{\rm G}^{},\rho_{\rm P}^{}$	Radii of profile curvature at pitch point in mean normal section	in	Eq C.31, C.30
ρ <sub>1</sub> , ρ <sub>2</sub>	Radii of profile curvature at point of maximum contact stress in mean normal section for pinion and gear, respectively	in	Eq C.36, C.37
$\Sigma_{\sf RN}$	Sum of gear and pinion mean normal pitch radii	in	Eq C.57
$\boldsymbol{\varphi}_{hG},\boldsymbol{\varphi}_{hP}$	Normal pressure angles at point of load application on the tooth centerline for gear and pinion, respectively		Eq C.63, C.62
$\phi_{LG}^{},\phi_{LP}^{}$	Normal pressure angles at point of load application on the tooth surface for gear and pinion, respectively		Eq C.59, C.58
ζ <sub>G</sub> , ζ <sub>Ρ</sub>	Angles between tangent of root fillet at weakest point and the centerline of tooth for gear and pinion, respectively		Eq C.90, C.77

# Table C.1 (concluded)

# C.2 Pitting resistance

# C.2.1 Pitting resistance geometry factor, I

The geometry factor, *I*, evaluates the relative radius of curvature of the mating tooth surfaces, and the load sharing between adjacent pairs of teeth at the point on the tooth surfaces where the calculated contact pressure will reach its maximum value.

# C.2.2 Graphs for geometry factor, I

Figures in Annex D contain graphs for the geometry factor, *I*, for straight bevel, zerol bevel and spiral bevel gears for a series of gear designs with a variety of pressure angles and spiral angles. These graphs may be used whenever the tooth proportions, face width, pressure angle and spiral angle correspond to those given on the graphs. Otherwise, Equation C.1 should be used.

# C.2.3 Formula for geometry factor, I

The *I* factor is calculated using the following formula:

$$I = \frac{s\rho_{\rm o} \cos\psi \cos\phi}{F \ d \ C_{\rm i} \ m_{\rm NI}} \frac{P_{\rm d}}{P_{\rm m}}$$

where

- s is length of line of contact, in (see C.1.3.4);
- $\rho_o$  is relative radius of profile curvature, in (see C.1.3.3);
- $\psi$  is mean spiral angle;
- $\phi$  is normal pressure angle;
- F is net face width, in. This is the narrower value of pinion and gear face widths;
- *d* is pinion outer pitch diameter, in;
- $C_i$  is inertia factor for gears with a low contact ratio, see C.1.3.5;

 $m_{\rm NI}$  is load sharing ratio per Equation C.42, see C.1.3.6;

- $P_{d}$  is outer transverse diametral pitch, in<sup>-1</sup>;
- $P_{\rm m}$  is mean transverse diametral pitch, in<sup>-1</sup>, per Equation C.8.

# C.2.3.1 Initial data

In order to solve the formula for geometry factor, *I*, the following additional data are required:

 $A_{o}$  is outer cone distance, in;

 $a_{\rm oP}$ ,  $a_{\rm oG}$  are outer addendums of pinion and gear, respectively, in;

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(C.1)

- *d*, *D* are outer pitch diameters of pinion and gear, respectively, in;
- *n*, *N* are numbers of pinion and gear teeth, respectively;
- $\gamma$ ,  $\Gamma$  are pitch angles of pinion and gear, respectively;
- $\gamma_o$ ,  $\Gamma_o$  are face angles of pinion and gear, respectively.

# C.2.3.2 Initial formulas

It will also be necessary to make the following calculations:

Mean cone distance, in

$A_{\rm m} = A_{\rm o} - 0.5F$	(C.2)
	( )

Pinion addendum angle

$$\alpha_{\mathsf{P}} = \gamma_{\mathsf{o}} - \gamma \tag{C.3}$$

Gear addendum angle

$$\alpha_{\rm G} = \Gamma_{\rm o} - \Gamma \tag{C.4}$$

Mean pinion addendum, in

$$a_{\rm P} = a_{\rm oP} - 0.5F \tan \alpha_{\rm P} \tag{C.5}$$

Mean gear addendum, in

$$a_{\rm G} = a_{\rm oG} - 0.5F \tan \alpha_{\rm G} \tag{C.6}$$

Location constant

$$k' = \frac{N - n}{3.2N + 4.0n} \tag{C.7}$$

Mean transverse diametral pitch, in-1

$$P_{\rm m} = \frac{A_{\rm o}}{A_{\rm m}} P_{\rm d} \tag{C.8}$$

Outer transverse circular pitch, in

$$p = \frac{\pi}{P_{\rm d}} \tag{C.9}$$

Mean normal base pitch, in

$$p_{\rm N} = \frac{A_{\rm m}}{A_{\rm o}} p \cos\psi \cos\phi \tag{C.10}$$

Mean normal circular pitch, in

$$p_{\rm n} = \frac{p_{\rm N}}{\cos\phi} \tag{C.11}$$

$$p_2 = \frac{p_n}{\cos\phi\left(\cos^2\psi + \tan^2\phi\right)} \tag{C.12}$$

Mean transverse pinion pitch radius, in

$$r = \frac{d}{2\cos\gamma} \frac{A_{\rm m}}{A_{\rm o}} \tag{C.13}$$

Mean transverse gear pitch radius, in

$$R = \frac{D}{2\cos\Gamma} \frac{A_{\rm m}}{A_{\rm o}} \tag{C.14}$$

Mean normal pinion pitch radius, in

$$r_{\rm N} = \frac{r}{\cos^2 \psi} \tag{C.15}$$

Mean normal gear pitch radius, in

$$R_{\rm N} = \frac{R}{\cos^2 w} \tag{C.16}$$

Mean normal pinion base radius, in

$$r_{\rm bN} = r_{\rm N} \cos \phi \tag{C.17}$$

Mean normal gear base radius, in

$$R_{\rm bN} = R_{\rm N} \cos\phi \tag{C.18}$$

Mean normal pinion outside radius, in

$$r_{\rm oN} = r_{\rm N} + a_{\rm P} \tag{C.19}$$

Mean normal gear outside radius, in

$$R_{\rm oN} = R_{\rm N} + a_{\rm G} \tag{C.20}$$

Length of mean normal pinion addendum action, in

$$Z'_{\rm P} = \sqrt{r_{\rm oN}^2 - r_{\rm bN}^2} - r_{\rm N} \sin\phi$$
(C.21)

Length of mean normal gear addendum action, in

$$Z'_{\rm G} = \sqrt{R_{\rm oN}^2 - R_{\rm bN}^2} - R_{\rm N} \sin\phi \tag{C.22}$$

Length of action in mean normal section, in

$$Z_{\rm N} = Z'_{\rm P} + Z'_{\rm G} \tag{C.23}$$

Transverse contact ratio. For straight bevel and zerol bevel gears this value must be greater than 1.0. Otherwise, these formulas are invalid.

$$m_{\rm p} = \frac{Z_{\rm N}}{p_2} \tag{C.24}$$

Face contact ratio. This value will be zero for straight bevel and zerol bevel gears.

$$K_{\mathsf{Z}} = \frac{F}{A_{\mathsf{o}}} \left[ \frac{2 - \frac{F}{A_{\mathsf{o}}}}{2\left(1 - \frac{F}{A_{\mathsf{o}}}\right)} \right]$$
(C.25)

$$m_{\rm F} = \frac{1}{\pi} \left( K_{\rm Z} \, \tan \psi - \frac{K_{\rm Z}^3}{3} \, \tan^3 \psi \right) A_{\rm o} P_{\rm d} \tag{C.26}$$

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Modified contact ratio

$$m_{\rm o} = \sqrt{m_{\rm p}^2 + m_{\rm F}^2}$$
 (C.27)

$$\cos\psi_{\rm h} = \cos\phi_{\rm v}/\cos^2\psi + \tan^2\phi$$
 (C.28)

where

 $\psi_{b}$  is mean base spiral angle.

$$\eta^{2} = Z_{N}^{2} \cos^{4} \psi_{b} + F^{2} \sin^{2} \psi_{b}$$
(C.29)

Mean normal pinion profile radius of curvature at pitch circle, in

$$\rho_P = \frac{r \sin \phi}{\cos^2 \psi_{\rm b}} \tag{C.30}$$

Mean normal gear profile radius of curvature at pitch circle, in

$$\rho_{\rm G} = \frac{R {\rm sin} \phi}{{\rm cos}^2 \psi_{\rm b}} \tag{C.31}$$

#### C.2.3.3 Determination of profile radius of curvature at critical point, $\rho_0$

The critical point on the tooth surface will occur when the line of contact passes through a point at a distance,  $f_{\rm l}$ , from the mid-point of the length of action,  $Z_{\rm N}$ . The value of  $f_{\rm l}$  is chosen to produce the minimum value of I, which corresponds to the point of maximum contact stress. For straight bevel and zerol bevel gears this line of contact will pass close to the lowest point of single tooth contact on the pinion, in which case:

$$f_1 = \frac{Z_N}{2} - p_N$$
 (C.32)

For spiral bevel gears:

 $f_{\rm I}$  = assumed value. For an initial value, use:

$$f_{\rm I} = 0 \tag{C.33}$$

Final value of  $f_1$  will usually lie between 0 and  $-\eta/2$ . Use steps of  $f_1$  equal to  $-\eta/10$  until *I* begins to increase, then use steps of  $+\eta/20$  until *I* begins to increase. Continue alternating the algebraic sign and halving the step until the magnitude of the step has been reduced to  $\eta/80$ . If the difference between the last two values of *I* is equal to or less than 0.001, use the smaller of these two values of *I*. Otherwise, continue to reduce the step of  $f_1$ .

$$\eta_{\rm I}^2 = \eta^2 - 4f_{\rm I}^2 \tag{C.34}$$

$$z_{\rm o} = \frac{Z_{\rm N}}{2} + \frac{Z_{\rm N}^2 f_{\rm l} \cos^2 \psi_{\rm b}}{\eta^2} + \frac{F Z_{\rm N} \eta_{\rm l} k' \sin \psi_{\rm b}}{\eta^2} - Z_{\rm G}'$$
(C.35)

Pinion profile radius of curvature at point  $f_{\rm l}$ , in

$$\rho_1 = \rho_P + z_o \tag{C.36}$$

Gear profile radius of curvature at point  $f_{\rm l}$ , in

$$\rho_2 = \rho_{\rm G} - z_{\rm o} \tag{C.37}$$

Relative radius of profile curvature, in

$$\rho_{0} = \frac{\rho_{1} \, \rho_{2}}{\rho_{1} + \rho_{2}} \tag{C.38}$$

# C.2.3.4 Length of line of contact, s

The length of the line of contact, in:

$$s = \frac{FZ_{\rm N}\eta \cos\psi_{\rm b}}{\eta^2} \tag{C.39}$$

### C.2.3.5 Inertia factor, C<sub>i</sub>

The inertia factor allows for the lack of smoothness of tooth action in dynamically loaded gears with a relatively low contact ratio, and is given by the following formula:

$$C_{\rm i} = \frac{2.0}{m_{\rm o}}$$
 when  $m_{\rm o} < 2.0$ , otherwise  $C_{\rm i} = 1.0$  (C.40)

For statically loaded gears such as those used in vehicle drive-axle differentials,  $C_i = 1.0$  even when  $m_o$  is less than 2.0.

# C.2.3.6 Load sharing ratio, m<sub>NI</sub>

The load sharing ratio is used to calculate the proportion of the total load carried on the tooth being analyzed, and is given by the following formulas:

$$\eta_{I}^{'3} = \eta_{I}^{3} + \sum_{k=1}^{k=a} \sqrt{\left[\eta_{I}^{2} - 4kp_{N}\left(kp_{N} + 2f_{I}\right)\right]^{3}} + \sum_{k=1}^{k=b} \sqrt{\left[\eta_{I}^{2} - 4kp_{N}\left(kp_{N} - 2f_{I}\right)\right]^{3}}$$
(C.41)

In the above formula, k is a positive integer which takes on successive values from 1 to a or b, generating all real terms (positive values under the radical) in each series. Imaginary terms (negative values under the radical) should be ignored. For most designs, a and b will not be greater than 2.

load sharing ratio

$$m_{NI} = \frac{\eta_I^3}{\eta_I^{\prime 3}}$$
 (C.42)

# C.3 Bending strength

# C.3.1 Bending strength geometry factor, J

The geometry factor, *J*, evaluates the shape of the tooth, the position at which the most damaging load is applied, the stress concentration due to the geometric shape of the root fillet, the sharing of load between adjacent pairs of teeth, the tooth thickness balance between the gear and mating pinion, the effective face width due to lengthwise crowning of the teeth, and the buttressing effect of an extended face width on one member of the pair. Both the tangential (bending) and radial (compressive) components of the tooth load are included.

#### C.3.2 Spiral bevel asymmetry

Since spiral bevel gear teeth are not symmetrical in the lengthwise direction, the stresses will differ between the concave and convex sides of the tooth. Normally one calculates the stresses on the concave side of the pinion tooth and convex side of the gear tooth since these are the usual driving surfaces. For bi-directional operation or in the case where the contact is pinion convex gear concave, an analysis calculating the geometry factors independently for the appropriate side of the tooth should be conducted unless appropriate test or experience indicates otherwise.

#### C.3.3 Graphs for geometry factor, J

Figures in Annex D contain graphs for the geometry factor, *J*, for straight bevel, zerol bevel, and spiral bevel gears for a series of gear designs. The graphs may be used whenever the tooth proportions, face widths, tooth thicknesses, tool edge radii, pressure angle, spiral angle, and driving surfaces correspond with those given on the graphs. Otherwise, the formulas in C.2.4 should be used.

# C.3.4 Formula for geometry factor, J

The J-factor is calculated using the following formulas:

Pinion geometry factor

$$J_{\mathsf{P}} = \frac{Y_{\mathsf{K}\mathsf{P}}}{m_{\mathsf{N}\mathsf{J}} K_{\mathsf{i}}} \frac{r_{\mathsf{t}}}{r} \frac{F_{\mathsf{e}\mathsf{P}}}{F_{\mathsf{P}}} \frac{P_{\mathsf{d}}}{P_{\mathsf{m}}}$$
(C.43)

Gear geometry factor

$$J_{\rm G} = \frac{Y_{\rm KG}}{m_{\rm NJ}K_{\rm i}} \frac{R_{\rm t}}{R} \frac{F_{\rm eG}}{F_{\rm G}} \frac{P_{\rm d}}{P_{\rm m}}$$
(C.44)

where

 $Y_{\text{KP}}$ ,  $Y_{\text{KG}}$  are tooth form factors including the stress concentration factor of pinion and gear, respectively, see C.2.4.8;

- $m_{\rm NJ}$  is load sharing ratio, see C.2.4.9;
- $K_i$  is inertia factor for gears with low contact ratio, see C.2.4.10;
- $r_t, R_t$  are mean transverse radii to point of load application for pinion and gear, respectively, in, see C.2.4.4;
- *r*, *R* are mean transverse pitch radii of pinion and gear, respectively, in, see C.13 and C.14;
- $F_{eP}$ ,  $F_{eG}$  are effective face widths of pinion and gear, respectively, in, see C.111 and C.116;
- $F_{P}$ ,  $F_{G}$  are the actual face widths of pinion and gear, respectively, in;
- $P_{d}$  is outer transverse diametral pitch, in<sup>-1</sup>;
- $P_{\rm m}$  is mean transverse diametral pitch, in<sup>-1</sup>, see C.8.

# C.3.4.1 Initial data

In order to solve the formula, the following data in addition to that given in C.1.3 and C.1.3.1 are required:

- $b_{oP}$ ,  $b_{oG}$  are outer dedendums of pinion and gear, respectively, in;
- $r_{\text{TP}}$ ,  $r_{\text{TG}}$  are tool edge radii of pinion and gear, respectively, in;
- $t_{\rm P}, t_{\rm G}$  are mean normal circular tooth thicknesses of pinion and gear, respectively, in;
- $\delta_{P}, \delta_{G}$  are dedendum angles of pinion and gear, respectively.

# C.3.4.2 Initial formulas

It will be necessary to calculate the following formulas in addition to those given in C.1.3.2.

mean pinion dedendum, in	
$b_{P} = b_{oP} - 0.5F \tan \delta_{P}$	(C.45)
mean gear dedendum, in	
$b_{\rm G} = b_{\rm oG} - 0.5F \tan \delta_{\rm G}$	(C.46)

# C.3.4.3 Determination of point of load application for maximum bending stress, $p_3$

For most straight bevel, zerol bevel and spiral bevel gears the maximum bending stress occurs at the equivalent of the highest point of single tooth contact when the modified contact ratio is equal to or less than 2.0. When the modified contact ratio is greater than 2.0, the line of contact is assumed to pass through the center of the path of action. For statically loaded straight bevel and zerol bevel gears such as those used in automotive differentials, the load is applied at the tip of the tooth. The position in any case is measured along the path of action from its center, and is designated by the symbol,  $f_{J}$ . Its distance from the beginning of the path of action is designated by the symbol  $p_{3}$ .

When  $m_0 \le 2.0$