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where

 $h_{\rm T}$  is total heat transfer coefficient, kW/m<sup>2</sup> – °K;

 $A_{T}$  is total surface area exposed to ambient air, m<sup>2</sup>;

$$\Delta T_{\rm S}$$
 is oil sump temperature rise, °C.

$$\Delta T_{\rm S} = T_{\rm sump} - T_{\rm A} \tag{60}$$

where

 $T_{sump}$  is oil sump temperature, °C;

 $T_A$  is ambient air temperature, °C.

$$h_{\rm T} = h_{\rm N} \left( 1 - \frac{A_{\rm F}}{A_{\rm T}} \right) + h_{\rm F} \left( \frac{A_{\rm F}}{A_{\rm T}} \right) + h_{\rm R} \tag{61}$$

where

 $h_{\rm N}$  is natural convection heat transfer coefficient, kW/m<sup>2</sup> – °K;

 $A_{\rm F}$  is surface area exposed to forced convection, m<sup>2</sup>;

 $h_{\rm F}$  is forced convection heat transfer coefficient, kW/m<sup>2</sup> – °K;

~ ~

 $h_{\rm B}$  is radiation heat transfer coefficient, kW/m<sup>2</sup> – °K.

$$h_{\rm N} = 0.0359 \left( D^{-0.1} \right) \left( \frac{\Delta T_{\rm S}}{T_{\rm A} + 273} \right)^{0.3} \tag{62}$$

where

*D* is outside diameter of largest drive ring gear, mm.

$$h_{\mathsf{F}} = 0.00705 \left( V^{0.78} \right) \tag{63}$$

where

V is cooling fan air velocity, m/s.

$$h_{\rm R} = \left(0.23 \times 10^{-9}\right) \varepsilon \left(\frac{T_{\rm sump} + T_{\rm A} + 546}{2}\right)^3 \tag{64}$$

where

ε is emissivity of drive outer surface.

Other cooling methods, such as an external heat exchanger, can be added to Equation 59 when required.

### 11.3.3 Heat generation

Per Equation 65, the heat generated in a planetary drive includes non-load dependent and load dependent losses. Non-load dependent losses,  $P_N$ , include contact oil seals,  $P_S$ , rolling bearing oil churning,  $P_{BO}$ , and gearing oil churning,  $P_{MO}$ .

$$P_{\rm N} = \sum P_{\rm S} + \sum P_{\rm BO} + \sum P_{\rm MO} \tag{65}$$

The load dependent losses,  $P_{\rm L}$ , include rolling bearing friction,  $P_{\rm BL}$ , gear mesh friction,  $P_{\rm ML}$ , and sleeve bearing friction,  $P_{\rm BS}$ .

$$P_{\rm L} = \sum P_{\rm BL} + \sum P_{\rm ML} + \sum P_{\rm BS} \tag{66}$$

Note that all contributors to each of the above sources of heat generation are summed to establish the total for the planetary gear drive. For a particular design or application other non-load dependent sources can be included in Equation 65, for example, shaft or motor driven oil pump. High speed planetary drives may need to consider windage losses.

### 11.3.3.1 Oil seals, P<sub>s</sub>

Lip type oil seals generate contact friction losses that depend on shaft speed, shaft size, oil sump temperature, oil viscosity, depth of oil seal immersion and the particular oil seal design. The following can be used to approximate the contact oil seal power loss for an individual seal [18]:

$$P_{\rm S} = \frac{C_{\rm S} \, D_{\rm S} \, n_{\rm SC}}{9549} \tag{67}$$

where

- $P_{\rm S}$  is contact oil seal power loss, kW;
- $C_{\rm S}$  is contact oil seal material constant;

 $C_{\rm S} = 0.003737$  for Viton

 $C_{\rm S} = 0.002429$  for Buna N

- $D_{\rm S}$  is diameter of shaft at oil seal contact, mm;
- $n_{\rm SC}$  is shaft speed at contact oil seal, rpm.

# 11.3.3.2 Rolling bearing oil churning, P<sub>BO</sub>

Bearing oil churning hydrodynamic losses depend on bearing speed, oil supply conditions, oil kinematic viscosity and size, and can be calculated per the following for an individual bearing [19]:

$$P_{\rm BO} = \frac{M_{\rm O} n_{\rm B}}{9549} \tag{68}$$

where

 $P_{\rm BO}$  is bearing churning power loss, kW;

 $M_{\rm O}$  is no-load torque of bearing, Nm;

 $n_{\rm B}$  is bearing rotational speed about its axis, rpm.

$$M_{\rm O} = 10^{-10} f_{\rm O} \left( \nu \, n_{\rm B} \right)^{0.667} d_{\rm M}^3 \tag{69}$$

where

 $f_{\rm O}$  is bearing dip factor;

- v is kinematic oil viscosity at oil sump temperature, mm<sup>2</sup>/s (centistokes);
- $d_{\rm M}$  is mean bearing diameter, mm.

$$d_{\rm M} = \frac{d_{\rm I} + d_{\rm O}}{2} \tag{70}$$

where

 $d_1$  is bearing bore diameter, mm;

 $d_{\rm O}$  is bearing outside diameter, mm.

The bearing dip factor,  $f_{O}$ , adjusts the torque based on the amount of bearing dip relative to the static oil level. Table 11 gives minimum (no dip) and maximum (submerged) values of  $f_{O}$  for various style bearings. For bearings on fixed axis shafts, linearly interpolate between the minimum and maximum  $f_{O}$  values based on the dip, H, relative to  $d_{M}$ :

$$f_{\rm O} = f_{\rm O\,min} + \frac{H}{d_{\rm M}} \left( f_{\rm O\,max} - f_{\rm O\,min} \right) \tag{71}$$

For planet gear bearings where the dip is not constant but varies with carrier rotation, the value of  $f_{O}$  should be selected based on the average of the smallest and largest  $f_{O}$  calculated relative to the static oil level.

This  $P_{BO}$  calculation does not apply to sealed bearings.

Bearing type	$f_{O(min)}$	$f_{O(max)}$
Deep groove ball bearings:		, ,
single-row	2	4
double-row	4	8
Self-aligning ball bearings	2	4
Angular contact ball bearings:		
single-row	3.3	6.6
double-row, paired single-row	6.5	13
Four-point contact ball bearings	6	12
Cylindrical roller bearings, with cage:		
series 10, 2, 3, 4	2	4
series 22	3	6
series 23	4	8
Cylindrical roller bearings, full complement:		
single-row	5	10
double-row	10	20
Needle roller bearings	12	24
Spherical roller bearings:		
series 213	3.5	7
series 222	4	8
series 223, 230. 239	4.5	9
series 231	5.5	11
series 232	6	12
series 240	6.5	13
series 241	7	14
Taper roller bearings:		
single-row	4	8
paired single-row	8	16
Thrust ball bearings	1.5	3
Cylindrical roller thrust bearings	3.5	7
Needle roller thrust bearings	5	11
Spherical roller thrust bearings:		
series 292 E	2.5	5
series 292	3.7	7.4
series 293 E	3	6
series 293	4.5	9
series 294 E	3.3	6.6
series 294	5	10

Table 11 – Bearing dip factor (oil bath lubrication),  $f_0$ 

# 11.3.3.3 Gearing oil churning, P<sub>MO</sub>

Gearing oil churning hydrodynamic losses in a planetary stage result from the rotation of the sun pinion, planet gear about its axis, and carrier. These losses depend on component speeds, oil supply conditions, oil kinematic viscosity, and size, and can be calculated per the following for an individual planetary stage [20], [21]:

$$P_{\rm MO} = P_{\rm CS} + P_{\rm CP} + P_{\rm CC}$$

where

 $P_{MO}$  is gearing oil churning power loss, kW;

 $P_{\rm CS}$  is churning loss of sun pinion, kW;

- $P_{CP}$  is churning loss of planet gears, kW;
- $P_{CC}$  is churning loss of carrier, kW.

(72)

(73)

$$P_{\rm CS} = \frac{A_{\rm C} f_{\rm S} \, \nu \, n_{\rm S}^3 \, d_{\rm OS}^{4.7} \, b_{\rm WS} \left(\frac{R_{\rm f}}{\sqrt{\tan\beta}}\right)}{10^{26}}$$

where

- $A_{\rm C}$  is carrier arrangement constant;
- $f_{\rm S}$  is sun pinion dip factor;
- $n_{\rm S}$  is sun pinion speed, rpm;
- $d_{OS}$  is sun pinion outside diameter, mm;
- $b_{WS}$  is sun pinion total face width, mm;
- $R_{\rm f}$  is roughness factor;
- β is generated helix angle, degrees; if less than 10 degrees, use 10 degrees in Equations 73 and 75. Use actual helix angle for all other equations.

$$R_{\rm f} = 7.93 - \frac{4.648}{m_{\rm t}} \tag{74}$$

where

 $m_{\rm t}$  is transverse tooth module, mm.

$$P_{\rm CP} = \frac{A_{\rm C} f_{\rm P} v n_{\rm P/C}^3 d_{\rm OP}^{4.7} b_{\rm WP} \left(\frac{R_{\rm f}}{\sqrt{\tan\beta}}\right)}{10^{26}} N_{\rm CP}$$
(75)

where

- $f_{\mathsf{P}}$  is planet gear dip factor;
- $n_{\rm P/C}$  is speed of planet gear relative to carrier, rpm;
- $d_{OP}$  is planet gear outside diameter, mm;
- $b_{WP}$  is planet gear total face width, mm;
- $N_{\rm CP}$  is number of planet gears.

$$P_{\rm CC} = \frac{A_{\rm C} f_{\rm C} v n_{\rm C}^3 D_{\rm C}^{4.7} W_{\rm C}}{10^{26}}$$
(76)

where

- $f_{\rm C}$  is carrier dip factor;
- $n_{\rm C}$  is carrier speed, rpm;
- $D_{\rm C}$  is carrier outside diameter, mm;
- $W_{\rm C}$  is carrier width, mm.

The sun pinion, planet gear and carrier dip factors are based on the amount of dip of each element relative to the static oil level. Since windage for industrial planetary drives is negligible with respect to other loses, the factor  $f_S = 0$  if the sun pinion does not dip,  $f_C = 0$  if the carrier does not dip, and  $f_P = 0$  if the planets do not dip. When the sun pinion is fully submerged in the oil,  $f_S = 1.0$ . When the carrier is fully submerged in the oil,  $f_P = 1.0$ . For a partially submerged element linearly interpolate between the dipping and non-dipping values. For example, if the sun pinion dips to its centerline,  $f_S = 0.5$ . The dip factor for the planet gear will be an average value based on the dip limits encountered in one revolution of the carrier.

The carrier arrangement constant is generally an empirical constant unique to a given planetary arrangement. It can be obtained from no-load thermal testing (see 11.2) by correlating measured oil sump temperature with the heat dissipation and non-load dependent loss math models presented in this standard.

## 11.3.3.4 Rolling bearing friction, P<sub>BL</sub>

The bearing friction loss depends on the coefficient of friction, load, size and speed. The bearing friction loss for an individual bearing [19] is given by:

$$P_{\rm BL} = \frac{\left(M_1 + M_2\right)n_{\rm B}}{9549} \tag{77}$$

where

 $P_{\rm BL}$  is bearing friction power loss, kW;

 $M_1$  is bearing load dependent friction torque, Nm;

 $M_2$  is bearing axial load dependent friction torque – cylindrical roller bearing only, Nm;

$$M_1 = \frac{f_1 P_1^a d_M^b}{1000} \tag{78}$$

where

 $f_1$  is bearing coefficient of friction, see Table 12;

*a*, *b* are exponents, see Table 13;

 $P_1$  is bearing dynamic load, N, see Table 12;

$$M_2 = \frac{f_2 F_a d_M}{1000}$$

where

 $f_2$  is bearing axial friction factor, see Table 14;

 $F_{a}$  is axial bearing load, N.

The values of  $f_2$  given in Table 14 assume adequate lubricant viscosity and that the ratio of axial to radial load does not exceed 0.50 for EC design (E = rollers added, C = open flange design [19]) and single row full complement bearings, 0.40 for other bearings with cage, or 0.25 for double row full complement bearings.

For tapered roller bearings, the induced axial thrust shall be considered in calculating the bearing dynamic load. Equations for calculating the proper  $F_a$  for use in Table 12 are given in Figure 26 for various bearing arrangements and load cases.

NOTE: Other methodology to calculate total bearing friction is available such as [20].

### 11.3.3.5 Hydrodynamic bearing loss

Shearing of the oil film in a sleeve bearing results in friction loss from both hydrodynamic and thrust washer sources [20]:

 $P_{\rm BS} = P_{\rm Bh} + P_{\rm Bt}$ 

(80)

(79)

where

 $P_{\rm BS}$  is sleeve bearing friction loss, kW;

 $P_{\rm Bh}$  is hydrodynamic sleeve bearing loss, kW;

 $P_{\rm Bt}$  is thrust washer power loss, kW.

Bearing type	$f_1$	$P_{1}^{(1)}$			
Deep groove ball bearings	$(0.0006 \dots 0.0009) (P_0/C_0)^{0.55 \ 2)}$	$3 F_{a} - 0.1 F_{r}$			
Self-aligning ball bearings	$0.0003 (P_0/C_0)^{0.4}$	$1.4 Y_2 F_a - 0.1 F_r$			
Angular contact ball bearings: single-row double-row, paired single-row	0.001 $(P_0/C_0)^{0.33}$ 0.001 $(P_0/C_0)^{0.33}$	$F_{a} - 0.1 F_{r}$ 1.4 $F_{a} - 0.1 F_{r}$			
Four-point contact ball bearings	$0.001 (P_0/C_0)^{0.33}$	$1.5 F_{a} + 3.6 F_{r}$			
Cylindrical roller bearings, with cage: series 10 series 2 series 3 series 4, 22, 23	0.000 2 0.000 3 0.000 35 0.000 4	$F_r^{3}$ $F_r^{3}$ $F_r^{3}$ $F_r^{3}$ $F^{3}$			
Cylindrical roller bearings, full	0.000 55	$F_r^{(3)}$			
Needle roller bearings	0.002	F			
Spherical roller bearings: series 213 series 222	0.000 22 0.000 15 0.000 65	If $F_r / F_a < Y_2 : 1.35 Y_2 F_a$			
series 230, 241 series 231 series 232	0.000 05 0.001 0.000 35 0.000 45	If $F_r / F_a \ge Y_2$ : $F_r \left[ 1 + 0.35 (Y_2 F_a / F_r)^3 \right]$			
series 239 series 240	0.000 25 0.000 8	(valid for all series)			
Taper roller bearings: single-row paired single-row	0.000 4 0.000 4	2 $YF_a^{4)}$ 1.2 $Y_2F_a^{4)}$			
Thrust ball bearings	0.000 8 $(F_a / C_0)^{0.33}$	F <sub>a</sub>			
Cylindrical roller thrust bearings, needle roller thrust bearings	0.001 5	F <sub>a</sub>			
Spherical roller thrust bearings: series 292E series 292 series 293E series 293 series 294E series 294	0.000 23 0.000 3 0.000 3 0.000 4 0.000 33 0.000 5	$F_{a}(F_{r} \max \leq 0.55F_{a})$ (valid for all series)			
Symbols: $P_0$ = equivalent static bearing load, N (see manufacturer's bearing tables); $C_0$ = basic static load rating, N (see manufacturer's bearing tables); $F_a$ = axial component of dynamic bearing load, N; $F_r$ = radial component of dynamic bearing load, N; $Y, Y_2$ = axial load factors (see manufacturer's bearing tables). <sup>1)</sup> If $P_1 < F_r$ , then $P_1 = F_r$ should be used.					
<sup>2)</sup> Small values are for light series bearings; large values for heavy series bearings.					

<sup>3)</sup> For bearings subjected to additional axial loads, refer to 11.3.3.4.
 <sup>4)</sup> Refer to 11.3.3.4.

Pooring type	Exponent	
bearing type	a	b
All (except spherical roller bearings)	1	1
Spherical roller bearings:		
series 213	1.35	0.2
series 222	1.35	0.3
series 223	1.35	0.1
series 230	1.5	-0.3
series 231, 232, 239	1.5	-0.1
series 240, 241	1.5	-0.2

# Table 13 – Exponents for calculation of $M_1$

Table <sup>-</sup>	14 –	Factor	f <sub>2</sub> for	cylindrical	roller	bearings
--------------------	------	--------	--------------------	-------------	--------	----------

	j	f <sub>2</sub> Lubrication		
Bearing	Lubri			
	grease	oil		
Bearings with cage:				
EC design	0.003	0.002		
other bearings	0.009	0.006		
Full complement bearings:				
single-row	0.006	0.003		
double-row	0.015	0.009		

Hydrodynamic sleeve bearing loss,  $P_{Bh}$ , can be estimated by the following equation:

$$P_{\rm Bh} = \frac{\mu_{\rm oil} \, n_{\rm B}^2 \, d_{\rm b}^3 \, L \, j \, 1.723 \times 10^{-17}}{c}$$

where

 $\mu_{oil}$  is absolute outlet oil viscosity, MPa·s;

 $d_{\rm b}$  is sleeve bearing bore, mm;

*L* is sleeve bearing contact length, mm;

*j* is bearing power loss coefficient, see Figure 27;

*c* is diametral clearance, mm.

The thrust washer power loss,  $P_{\rm Bt}$ , is:

$$P_{\rm Bt} = \frac{\mu_{\rm oil} \, n_{\rm B}^2 \left(r_{\rm o}^4 - r_{\rm i}^4\right) 1.723 \times 10^{-17}}{t} \tag{82}$$

where

*r*<sub>o</sub> is outside radius of thrust washer, mm;

*r*<sub>i</sub> is inside radius of thrust washer, mm;

*t* is oil film thickness, mm.

The Sommerfield Number, S, used in Figure 27 is calculated by the following:

$$S = \frac{d_b^2 \,\mu_{\text{oil}} \,n_{\text{B}} \times 10^{-6}}{c^2 \,w \,60} \tag{83}$$

where

w is load per unit area, kPa

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(81)

Arrangement	Load case	Axial loads
B Back-to-back A	1a) $\frac{F_{rA}}{Y_A} \ge \frac{F_{rB}}{Y_B}$ $K_a \ge 0$	$F_{aA} = \frac{0.5F_{rA}}{F_{aB} = F_{aA} + K_{a}}$ $F_{aB} = F_{aA} + K_{a}$
F <sub>rB</sub> F <sub>rA</sub> A Face-to-face B	1b) $\frac{F_{rA}}{Y_A} < \frac{F_{rB}}{Y_B}$ $K_a \ge 0.5 \left(\frac{F_{rB}}{Y_B} - \frac{F_{rA}}{Y_A}\right)$	$F_{aA} = \frac{0.5F_{rA}}{Y_{A}}$ $F_{aB} = F_{aA} + K_{a}$
K <sub>a</sub> F <sub>r</sub> A F <sub>r</sub> B	1c) $\frac{F_{rA}}{Y_{A}} < \frac{F_{rB}}{Y_{B}}$ $K_{a} < 0.5 \left(\frac{F_{rB}}{Y_{B}} - \frac{F_{rA}}{Y_{A}}\right)$	$F_{aA} = F_{aB} - K_{a}$ $F_{aB} = \frac{0.5F_{rB}}{Y_{B}}$
A Face-to-face B	2a) $\frac{F_{rA}}{Y_A} \le \frac{F_{rB}}{Y_B}$ $K_a \ge 0$	$F_{aA} = F_{aB} + K_{a}$ $F_{aB} = \frac{0.5 F_{rB}}{Y_{B}}$
F <sub>rA</sub> F <sub>rB</sub> B Back-to-back A	2b) $\frac{F_{rA}}{Y_{A}} > \frac{F_{rB}}{Y_{B}}$ $K_{a} \ge 0.5 \left(\frac{F_{rA}}{Y_{A}} - \frac{F_{rB}}{Y_{B}}\right)$	$F_{aA} = F_{aB} + K_{a}$ $F_{aB} = \frac{0.5 F_{rB}}{Y_{B}}$
<i>K</i> a <i>F</i> rB <i>F</i> rA	2c) $\frac{F_{rA}}{Y_A} > \frac{F_{rB}}{Y_B}$ $K_a < 0.5 \left(\frac{F_{rA}}{Y_A} - \frac{F_{rB}}{Y_B}\right)$	$F_{aA} = \frac{0.5 F_{rA}}{Y_{A}}$ $F_{aB} = F_{aA} - K_{a}$

Figure 26 – Tapered roller bearing load equations



Figure 27 – Bearing power loss coefficient, j

#### 11.3.3.6 Gear friction, P<sub>ML</sub>

Gear tooth friction loss is a function of the mechanics of tooth action, coefficient of friction, speed and transmitted torque. Tooth action involves relative sliding between meshing teeth separated by an oil film. The coefficient of friction depends on lubricant properties, load intensity and speed. Mesh friction power loss [20], [21] for an individual planetary stage can be expressed by:

$$P_{\mathsf{ML}} = \left(P_{\mathsf{MLE}} + P_{\mathsf{MLI}}\right) N_{\mathsf{CP}} \tag{84}$$

where

 $P_{\rm ML}$  is total planetary stage mesh friction power loss, kW;

 $P_{MLE}$  is friction power loss at sun/planet (external) mesh, kW;

 $P_{MLI}$  is friction power loss at planet/ring (internal) mesh, kW.

$$P_{\mathsf{MLE}} = \frac{f_{\mathsf{e}} T_{\mathsf{e}} n_{\mathsf{S/C}} \cos^2 \beta_{\mathsf{we}}}{9549 M_{\mathsf{e}}} \tag{85}$$

where

 $f_{\rm e}$  is external mesh coefficient of friction;

 $T_{\rm e}$  is sun pinion torque per mesh;

 $n_{S/C}$  is speed of sun gear relative to carrier, rpm;

 $\beta_{we}$  is sun/planet operating helix angle, degree;

 $M_{\rm e}$  is external mesh mechanical advantage.

$$f_{\rm e} = \frac{v^{-0.223} K_{\rm e}^{-0.40}}{3.239 V_{\rm e}^{0.70}} \tag{86}$$

where

v is kinematic oil viscosity at oil sump temperature,  $mm^2/s$ ;

 $K_{\rm e}$  is external mesh load intensity, N/mm<sup>2</sup>;

 $V_{\rm e}$  is sun/planet pitchline velocity, m/s.

$$K_{\rm e} = \frac{1000T_{\rm e}(z_{\rm S} + z_{\rm P})}{2b_{\rm we} r_{\rm wS}^2 z_{\rm P}}$$
(87)

where

- $z_{\rm S}$  is number of sun pinion teeth;
- $z_{P}$  is number of planet gear teeth;
- $b_{\rm we}$  is engaged sun/planet face width, mm;
- $r_{\rm wS}$  is sun pinion operating pitch radius, mm.

$$M_{\rm e} = \frac{2\cos\alpha_{\rm we} \left(H_{\rm se} + H_{\rm te}\right)}{H_{\rm se}^2 + H_{\rm te}^2} \tag{88}$$

where

 $\alpha_{we}$  is sun/planet mesh transverse operating pressure angle, degree;

 $H_{se}$  is sun/planet mesh sliding ratio at start of approach;

 $H_{\text{te}}$  is sun/planet mesh sliding ratio at end of recess.

$$H_{\rm se} = \left(u_{\rm e} + 1\right) \left( \left(\frac{r_{\rm oP}^2}{r_{\rm wP-S}^2} - \cos^2 \alpha_{\rm we}\right)^{0.5} - \sin \alpha_{\rm we} \right)$$
(89)

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where

- $u_{\rm e}$  is planet/sun tooth ratio;
- $r_{oP}$  is planet gear outside radius, mm;

 $r_{wP-S}$  is planet-sun gear operating pitch radius, mm.

$$u_{\mathsf{e}} = \frac{z_{\mathsf{P}}}{z_{\mathsf{S}}} \tag{90}$$

$$H_{\text{te}} = \left(\frac{u_{\text{e}} + 1}{u_{\text{e}}}\right) \left( \left(\frac{r_{\text{oS}}^2}{r_{\text{wS}}^2} - \cos^2 \alpha_{\text{we}}\right)^{0.5} - \sin \alpha_{\text{we}} \right)$$
(91)

where

 $r_{oS}$  is sun pinion outside radius, mm.

$$P_{\mathsf{MLI}} = \frac{f_{\mathsf{i}} T_{\mathsf{i}} n_{\mathsf{P/C}} \cos^2 \beta_{\mathsf{wi}}}{9549 M_{\mathsf{i}}} \tag{92}$$

where

- $f_i$  is internal mesh coefficient of friction;
- $T_i$  is planet gear torque, Nm;
- $\beta_{wi}$  is planet/ring operating helix angle, degree;
- $M_{\rm i}$  is internal mesh mechanical advantage.

$$f_{\rm i} = \frac{v^{-0.223} K_{\rm i}^{-0.40}}{3.239 V_{\rm i}^{0.70}} \tag{93}$$

where

- $K_i$  is internal mesh load intensity, N/mm<sup>2</sup>;
- $V_{i}$  is planet/ring pitchline velocity, m/s.

$$K_{\rm i} = \frac{1000 T_{\rm i} (z_{\rm R} - z_{\rm P})}{2 b_{\rm wi} r_{\rm wP-R}^2 z_{\rm R}}$$
(94)

where

 $z_{\rm R}$  is number of ring gear teeth;

 $b_{wi}$  is engaged planet/ring face width;

 $r_{\rm wP-R}$  is planet-ring gear operating pitch radius, mm.

$$M_{i} = \frac{2\cos\alpha_{wi}(H_{si} + H_{ti})}{H_{si}^{2} + H_{ti}^{2}}$$
(95)

where

- $\alpha_{wi}$   $\;$  is planet/ring mesh transverse operating pressure angle, degree;
- $H_{si}$  is planet/ring mesh sliding ratio at start of approach;
- $H_{\rm ti}$  is planet/ring mesh sliding ratio at end of recess.

$$H_{\rm si} = \left(u_{\rm i} - 1\right) \left[\sin\alpha_{\rm wi} - \left(\frac{r_{\rm iR}^2}{r_{\rm wR}^2} - \cos^2\alpha_{\rm wi}\right)^{0.5}\right]$$
(96)