

# **Improved Tooth Load Distribution in an Involute Spline Joint Using Lead Modifications Based on Finite Element Analysis**

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**American Gear Manufacturers Association**



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## **TECHNICAL PAPER**

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**Frederick W. Brown, Jeffrey D. Hayes and G. Keith Roddis, The Boeing Company**

[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

## **Abstract**

Involute splines in torque transmitting joints are prone to non-uniform contact loading along their length especially in lightweight, relatively flexible applications such as a helicopter main rotor shaft-to-rotor hub joint. The structural stiffness and internal load paths of the two members in the joint affect spline tooth contact pressure distribution. In such applications, in the absence of lead corrections, the torque is transferred non-uniformly along the length of the spline resulting in a concentration or peaking of the tooth contact load at one end of the spline.

A significantly improved tooth load distribution was achieved for splines for the Low Maintenance Rotor (LMR) version of the CH-47 Chinook helicopter main rotor shaft-to-rotor hub joint by applying, to the internally splined member, complex lead corrections which varied continuously along the length of the spline. The required lead corrections were determined analytically using finite element methods (FEM). Rotor hub splines with the analytically determined lead corrections were manufactured and tested under design load conditions. A standard CH-47 rotor shaft-to-hub joint, which uses a step lead correction between splines, was previously tested. Strain gages were used to infer contact load distribution along the length of the splines.

Test data indicated that the complex lead corrections resulted in a nearly uniform contact load distribution along the length of the spline at the design torque load. The data also showed that the load distribution for the splines with the complex lead corrections was significantly improved relative to the contact load distribution of the baseline splines. This work was performed under the U.S. Army Aviation and Missile Command (AMCOM) Low Maintenance Rotor (LMR) hub development contract DAAH01-99-3-R001.

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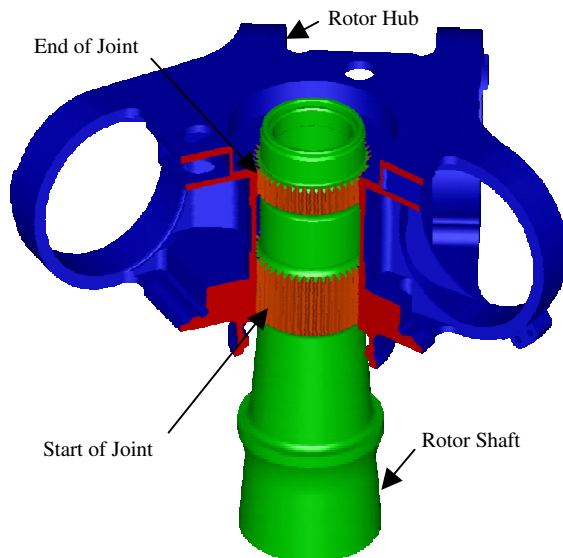
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## Introduction

Involute splines offer a compact and weight-efficient means of transferring torque from one shaft to another, or between a shaft and a hub. Involute spline tooth dimensions and tolerances have been standardized by the Society of Automotive Engineers, American Society of Mechanical Engineers and others, and are published in reference [1]. The basic equations for involute spline tooth stress calculations assume that spline tooth loading is uniformly distributed along the length of the spline tooth. Non-uniform tooth loading is addressed in some spline load rating calculations by applying a "load distribution" factor as in [2] and [3]. Load distribution factors are used to account for misalignment (slope) between the internal and external spline members. The load distribution factor is influenced by the magnitude of the misalignment (slope)

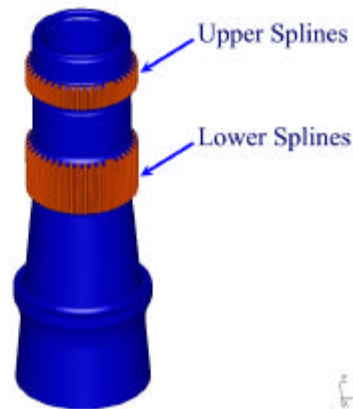
between the members, and by crowning of the spline teeth which reduces end-loading of the spline teeth thereby accommodating the misalignment.

There is another mechanism that can cause non-uniform contact loading of spline teeth. It has been reported in [4], that as the length of a spline increases relative to its diameter, the torsional stiffness of the members in the joint exert a stronger influence on the lengthwise contact load distribution. This non-uniform contact load distribution can occur in perfectly aligned spline joints. The mechanism is not dependent on angular misalignment of the member's axes, but rather by the relative torsional stiffnesses and deflections (wind-up) of the internal and external members. In longer splines, the tooth contact load peaks near the start of the joint then falls away toward the end of the joint [4]. The start of the joint is considered to be where torque first begins to be transferred from the inner member to the outer member, as in a shaft to a hub. In splined joints where the inner and outer members have complex geometries (rather than simple cylindrical geometries) torsional stiffness can vary non-uniformly along the length of the joint leading to further non-uniformity of the tooth contact load distribution. Indeed, in some splined members with complex geometries, very stiff "hard points" may exist that resist torsional deflection and result in high contact loads over relatively short tooth lengths. Predicting tooth contact load distribution for these situations can be quite difficult. One application that utilizes a relatively long splined joint, between a shaft and a hub with complex geometries, occurs on the CH-47 helicopter. The splined joint in question transfers torque and rotary motion from the rotor shaft to the rotor hub. The rotor hub provides the attachment (via lugs) and load transfer to the helicopter rotor blades. A sectioned view of the rotor hub with the



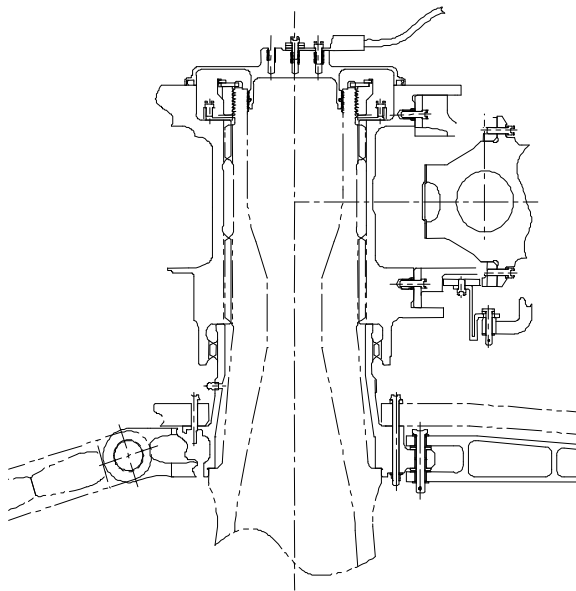
**Figure 1 LMR Rotor Hub and Shaft  
(Section of Hub Removed for Clarity)**

mating shaft is shown in Figure 1. The shaft spline teeth that mate with the hub are split into two lengths, an upper and lower spline, with an un-splined cylindrical section between them as shown in Figure 2.



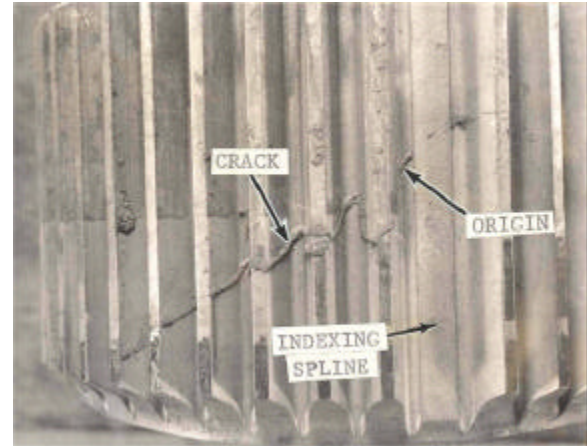
**Figure 2 CH-47 Rotor Shaft**

A section-view through the members is shown in Figure 3. The splined joint in this application is fixed and preloaded. The bending loads generated in the rotor hub are transferred primarily by the “clamp-up” between the hub and a shoulder on the rotor shaft, rather than through bearing on the spline teeth. The spline teeth act to transfer torque between the rotor shaft and hub, in turn, driving the main rotor blades.



**Figure 3 Cross-Section Through Rotor Shaft/Hub Splined Joint**

In this application, the splined joint has functioned well for many years. However, it has been shown in fatigue testing to be one of the critical sections of the rotor shaft (inner member) and is an impediment to further torque growth. During fatigue testing, torsional fatigue loads are increased (overloaded) until fatigue cracking occurs. A torsional fatigue crack in the splined area of the rotor shaft, from fatigue overload testing, is shown in Figure 4.



**Figure 4 Shaft fatigue test specimen showing a crack at the spline**

Of course it took loads much greater in magnitude than actual aircraft loads to generate this failure. There is evidence, such as spline wear patterns, that indicate the pressure distribution along the splines is highly non-uniform.

The current splined joint configuration has a simple “step” modification to reduce load peaking at the lower end of the lower spline. This “step” modification is applied to the hub (internal spline) member. The “step” modification was designed to cause the upper spline to carry a larger portion of the total torque transmitted by the joint. The “step” modification effectively indexes the lower spline teeth relative to the upper spline teeth and was accomplished by thinning the lower spline teeth. No lead modifications are applied to the upper and lower spline teeth themselves, but the upper spline is indexed relative to the lower spline. The indexing results in a 0.005-0.006 inch tangential position difference between drive flanks of the upper and lower spline teeth.