Aluminum Design Manual

PART II-B

Commentary on Specification for Aluminum Structures Load and Resistance Factor Design



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General Introduction

This Commentary is not intended to provide a general primer to probability-based Load and Resistance Factor Design (LRFD) criteria. This is provided in Reference (2) and the further references cited therein. The purpose of this commentary is to give an explanation for the reasons for the recommended resistance factors in Part IB, Load and Resistance Factor Design of Buildings and Similar Type Structures.

Section 2.3 Loads

Factored load combinations for building type structures given in ASCE 7-02 are:

1) 1.4(D + F)

- 2) $1.2(D + F + T) + 1.6(L + H) + 0.5(L_r \text{ or } S \text{ or } R)$
- 3) $1.2D + 1.6 (L_r \text{ or } S \text{ or } R) + (L \text{ or } 0.8W)$
- 4) $1.2D + 1.6W + L + 0.5(L_r \text{ or } S \text{ or } R)$
- 5) 1.2D + 1.0E + L + 0.2S
- 6) 0.9D + 1.6W + 1.6H
- 7) 0.9D + 1.0E + 1.6H

Exceptions:

- 1. The load factor on *L* in combinations (3), (4), and (5) is permitted to equal 0.5 for all occupancies in which *L* is less than or equal to 100 psf, with the exception of garages or areas of public assembly.
- 2. The load factor on *H* shall be set equal to zero in combinations (6) and (7) if the structural action due to *H*

Section 3. General Design Rules

The general procedure of applying the Load and Resistance Factor Design (LRFD) method for aluminum building structures consists of the following steps:

- 1) Determine the stress due to the factored loads, *f*, by conventional elastic structural analysis. The factored loads are the code-specified dead, live, wind, rain, snow or earthquake loads multiplied by the load factors given in Section 2.3.
- 2) Compute the factored limit state stress $\phi F_{\rm L}$ from Section 3.4 and verify that $\phi F_{\rm L} \ge f$

Section 3.4 gives the resistance factor ϕ and the limit state stress F_L for a variety of commonly encountered aluminum structural members and elements. The limit state stress F_L is dependent on the material properties and the member geometry. It reflects the ultimate load carrying capacity of the member or element, be that yield, fracture, plastification, buckling or crippling. The limit state stresses counteracts that due to W or E. Where lateral earth pressure provides resistance to structural actions from other forces, it shall not be included in H but shall be included in the design resistance.

where

D = dead load

- E = earthquake load
- F = loads due to fluids with well-defined pressures and maximum heights
- H = load due to lateral earth pressure, ground water pressure, or pressure of bulk materials
- L = live load
- $L_r = \text{roof live load}$
- R = rain load
- S = snow load
- T = self-straining force
- W =wind load

in these LRFD criteria are identical to those which are given in the ASD *Specification for Aluminum Structures*. They can be determined simply by setting the factors of safety equal to unity in the various formulas given in Section 3.4 of Part IA.

The resistance factor ϕ accounts for the uncertainties of determining the limit state stress. It is computed by the method of first-order second-moment probabilistic analysis presented in Reference (2) for a target reliability index of $\beta_T = 2.5$ for the yield limit state and $\beta_T = 3.0$ for the fracture limit state. Following is a detailed account presenting the background for each of the resistance factors used in Section 3.4 of the LRFD criteria.

Prior to this detailed account it will be instructive to discuss in a simple manner the basic concepts of probabilistic design. Failure is defined when the resistance, as characterized by a limit state, is less than or equal to the load effect on the structural element. The load effect in these LRFD criteria for aluminum structures is characterized by the

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stress computed by elastic analysis from the forces acting on the structure. Both the resistance R and the load effect Qare random quantities (Fig.C1).

Limit states are either ultimate or serviceability limit states. These LRFD criteria pertain to the ultimate limit states of yield, fracture, plastification, buckling and crippling, although the serviceability limit states of deflection and the appearance of buckling are also featured (in Section 4).

Failure is then not necessarily the total collapse of the member, but the reaching of a practically defined ultimate limit state. It occurs when R < Q. Alternately, failure also is defined as in $\ln(R/Q) \le 0$, as shown in Fig.C2. The probability of exceeding a limit state is the shaded area. According to present practice, it is not necessary to define a desired probability of failure, but a "reliability index" β is determined such that the "target reliability index" $\beta_{\rm T}$ for a new code is approximately equal to the value of β inherent in the traditional specification for standard design situations (2). This process of selecting a target reliability index is called "code calibration." It will be illustrated for the simple case of tension members.

According to first-order statistical derivations, the value of β from Fig.C2 is expressed by the following formula.

$$\beta = \frac{\ln(\overline{R}/\overline{Q})}{\sqrt{V_R^2 + V_Q^2}} \tag{1}$$

In this equation \overline{R} and \overline{Q} and are the mean values of the resistance R and the load effects Q, respectively, and V_R and V_Q are the corresponding coefficients of variation.

The resistance of a tension member for the limit state of yielding is

$$R = A F_{ty} \tag{2}$$

and thus

$$\overline{R} = \overline{A}\overline{F}_{ty} \tag{3}$$

PROBABILITY DENSITY



Figure C-1 SCHEMATIC REPRESENTATION OF PROBABILITIES OF THE LOAD EFFECT AND THE RESISTANCE



and

$$V_R = \sqrt{V_A^2 + V_{Fty}^2} \tag{4}$$

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The available data on dimensions and yield stress of aluminum structures were evaluated in Reference (3), and the following conservative estimates of the statistical properties were suggested:

$$\overline{F}_{ty} = 1.10F_{tyn}, V_{Fty} = 0.06, \overline{A} = A_n, V_A = 0.05$$

where F_{tyn} is the minimum specified yield stress and A_n is the handbook area. These are the "nominal" values the designer uses. With these values

$$\overline{R} = 1.10R_n$$
 and $V_R = \sqrt{0.05^5 + 0.06^2} = 0.08$
 R_n is the "nominal" resistance, $R_n = A_n F_{tyn}$.

The load effect Q is the tensile force in the member due to the applied loads. For purposes of illustration only dead and live load will be used, i.e.,

$$Q = D + L \tag{5}$$

$$\overline{Q} = \overline{D} + \overline{L} \tag{6}$$

$$V_{\mathcal{Q}} = \frac{\sqrt{(\overline{D}V_D)^2 + (\overline{L}V_L)^2}}{\overline{D} + \overline{L}}$$
(7)

The following statistical data about load are taken from Reference (2):

 $\overline{D} = 1.05D_n, \overline{L} = L_n, V_D = 0.1, V_L = 0.25$

where D_n and L_n are the "nominal", code specified, loads.

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Rearrangement of Eqs. 6 and 7 leads to the following equations:

$$\overline{Q} = L_n \left(1.05 \ D/L + 1 \right) \tag{8}$$

$$V_{Q} = \frac{\sqrt{(1.05 \times D/L)^{2} + 0.25^{2}}}{1.05 D/L + 1}$$
(9)

where D/L is the nominal dead-to-live load ratio.

The process of calibrating to the ASD *Specification* is performed as follows:

$$A_n F_{tyn}/F.S. = D_n + L_n \tag{10}$$

or

$$R_n = F.S. \ (D_n + L_n) = F.S. \ (L_n)(D/L + 1)$$
(11)

F.S. is the specified factor of safety, which is equal to 1.65 in the ASD *Specification* for the limit state of yield.

Substitution of *F.S.* = 1.65 into Eq. 11, and use of Eq. 11 in the relationship $\overline{R}/\overline{Q}$ gives

$$\frac{\overline{R}}{\overline{Q}} = \frac{1.0 \times 1.65 \ (D/L+1)}{1.05 \ D/L+1} \tag{12}$$

 $\overline{R}/\overline{Q}$ and V_Q (Eq. 9), and thus also β (Eq. 1), depend on the dead-to-live load ratio. Aluminum structures usually have a low dead-to-live load ratio. Following are values of β determined from Eq. 1 for the limit state of yield (*F.S.* = 1.65) and the limit state of fracture (*F.S.* = 1.95). For this latter case $\overline{R} = 1.10 R_n$ and $V_R = 0.08$, as for the limit state of yield (Reference 3).

D/L	ß Yield	ß Fracture
0.2	2.6	3.4
0.1	2.5	3.2

A similar exercise can also be performed for the proposed LRFD method. According to this approach

$$\phi A_n F_{tyn} = \gamma_D D_n + \gamma_L L_n \tag{13}$$

Again, using $R_n = A_n F_{tyn}$, and $\gamma_D = 1.2$ and $\gamma_L = 1.6$ as recommended in Reference (2),

$$R_n = \frac{L_n}{\phi} (1.2 \ D/L + 1.6) \tag{14}$$

from which

$$\overline{R}/\overline{Q} = \frac{1.10}{\phi} \left[\frac{1.2 \ D/L + 1.6}{1.05 \ D/L + 1} \right]$$
(15)

The calculations show the following results:

		ß	D/L	ϕ
limit state viold)	2.5	0.2	0.95
minit state yield	J	2.5	0.1	0.95
limit stata fractura)	3.1	0.2	0.85
minit state macture	}	2.9	0.1	0.85

The values of ϕ were rounded off to the nearest 0.05, and comparison of the β 's indicates that for typical deadto-live load ratios of aluminum structures (i.e., D/L of 0.2 to 0.1) the values of β are near the target of 2.5 for the limit state of yield, and the target of $\beta_T = 3.0$ for the fracture limit state. This difference reflects the fact of the greater reliability demanded for the more serious type of limit state, as already recognized in the ASD *Specification* with its two kinds of safety factors, i.e., 1.65 and 1.95. These target reliability indices are similar to those used by the AISI for cold-formed steel.

Based on the results presented above $\phi = 0.95$ is recommended for the limit state of yield, and $\phi = 0.85$ for the limit state of fracture. Methods are available to easily check the consequences of changing ϕ as regards reliability. The economic consequences can also be ascertained by comparing designs required by the ASD and the LRFD method, as follows:

$$(R_n)_{ASD} = L_n (D/L + 1)(F.S.)$$
(16)

$$(R_n)_{LRFD} = L_n (1.2 D/L + 1.6)(1/\phi)$$
(17)

when $(R_n)_{ASD}$ is the nominal design requirement according to Part I-A, and $(R_n)_{LRFD}$ is the requirement of the LRFD criteria. The ratio *LRFD/ASD* is then

$$\frac{1.2 D/L + 1.6}{\phi (F.S.) (D/L + 1)}$$
(18)

The curves in Fig. C-3 show the variation of this ratio for various values of ϕ and for F.S. = 1.65 and 1.95 for the range D/L = 0.2 to 0.5. It can be seen that the ratio decreases with an increase of the dead-to-live load ratio.

The following portions of this commentary will give the basic data used to arrive at the recommended ϕ -factors in Section 3.4.

3.4.1 Tension, Axial

The selection of $\phi_y = 0.95$ and $\phi_u = 0.85$ was discussed in the previous part of this Commentary.

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THE EFFECTS OF CHANGING THE RESISTANCE FACTOR Φ ON THE REQUIRED AREA FOR TENSION MEMBERS

3.4.2 through 3.4.4 Tension in Extreme Fibers of Beams

Two limit states apply to the tension flange: limit state of yield when the strain is that corresponding to the yield stress F_{ty} , and limit state of fracture. The resistance is the bending moment M, and its mean value and coefficient of variation is, for the yield limit state,

$$\overline{R} = \overline{S_{xt}} \,\overline{g} \,\overline{F_{ty}} \tag{19}$$

and

$$V_R = \sqrt{V_{S_u}^2 + V_g^2 + V_{F_v}^2}$$
(20)

where S_{xt} is the elastic section modulus on the tension side, g is the "shape factor", and F_{ty} is the tensile yield stress. The same expressions hold for the limit state of fracture, with the exception that F_{ty} is replaced by F_{tu} . The shape factor accounts for partial plastification due to the non-linear nature of the stress-strain curves. The nominal resistance is

$$R_N = S_{xtn} g_n F_{tyn} \tag{21}$$

and so

$$\overline{R} = R_n = \left(\frac{\overline{S}_{xt}}{S_{xtn}}\right) \left(\frac{\overline{g}}{g_n}\right) \left(\frac{\overline{F}_{ty}}{F_{tyn}}\right)$$
(22)

Reference (3), as noted before for the tension member, gives the values

$$\overline{S}_{xt} = S_{xt}, V_{Sxt} = 0.05, \overline{F}_{ty} = 1.10F_{tyn}, V_{Fty} = 0.06$$

It will be assumed that g_n equals the shape factors in Part I-A, and equals the values given in Reference (4), which were also corroborated for some sections and alloys in Reference (5). It will be assumed that $V_g = 0.0$. From these data \overline{R} and V_R can be determined as

$$\overline{R} = R_n (1.1\overline{g}/g_n)$$
 and $V_R = \sqrt{0.05^5 + 0.06^2} = 0.08$

The results of the analysis for the recommended ϕ -factors are given in Table C-3.4.1. The values of β are near the target values.

3.4.5 and 3.4.6 Bearing

In the absence any statistically significant data on bearing capacities, it was decided to use $\phi_u = 0.85$, giving essentially the same requirements as the ASD *Specification*.

3.4.7 Compression in Columns, Axial, Gross Section

The nominal column strength equations of the ASD *Specification* were retained, i.e.,

$$F_L = B_c - D_c \, kL/r \le F_{cy} \tag{23}$$

for $kL/r \le S_2 = C_c$, and

$$F_L = \frac{\pi^2 E}{(kL/r)^2} \tag{24}$$

for $kL/r \ge C_c$

It was found convenient in the background research to introduce a non-dimensional slenderness ratio

$$\lambda = \frac{kL}{r} \left(\frac{1}{\pi}\right) \sqrt{F_{cy}/E} \tag{25}$$

and the equations actually given in Section 3.4.7 are in terms of λ rather than the effective slenderness ratio. The definitions of B_c , D_c , S_2 and C_c remain the same as in Part I-A. The relationship between the nominal limit state stress F_L and the factored limit state stress ϕF_L , and the slenderness parameter λ , is shown in Fig. C-4 for one particular alloy.

The resistance factor ϕ_{cc} varies with the slenderness parameter. The particular equation for ϕ_{cc} given in Section 3.4.7 is similar to, but not identical to, the resistance factors recommended in References (3) and (5), where considerable work was done in the development of LRFD provisions for columns, and therefore, a detailed accounting is presented on the way ϕ_{cc} was selected. The mean resistance of an ideally pinned-end but initially crooked column was shown to be equal to (3, 5):

$$\overline{R} = \overline{A} \ \overline{\sigma}_{TM} \ \overline{B}_T \ \overline{B}_u \tag{26}$$

The coefficient of variation is then

$$V_R = \sqrt{V_A^2 + V_{\sigma_{TM}}^2 + V_{B_T}^2 + V_{B_u}^2}$$
(27)

The terms in Eq. 26 are defined as follows:

\overline{A} : mean cross-sectional area of column

In accordance with previous usage, $\overline{A} = A_n$ and $V_A = 0.05$, where A_n is the nominal area.

 σ_{TM} : mean buckling stress of an ideally straight column as determined by the tangent modulus theory, i.e.,

$$\sigma_{TM} = \frac{\pi^2 E_t}{(kL/r)^2} \tag{28}$$

In the derivation of References (3) and (5) a Ramberg-Osgood type stress-strain curve was assumed, and thus the tangent modulus E_t is equal to

$$E_{t} = \frac{E}{1 + 0.002n \left(\frac{E}{\sigma_{0.2}}\right) \left(\frac{\sigma}{\sigma_{0.2}}\right)^{n-1}}$$
(29)

In this equation *E* is the elastic modulus, σ is the average stress under this buckling load, $\sigma_{0.2}$ is the compressive stress when the strain is equal to 0.2 percent, and *n* is the strain-hardening parameter. The coefficient of variation of σ_{TM} , $V\sigma_{TM}$, was shown to be 0.06 in Reference (5).

Cross Section and Flexure Plane	Article in LRFD Criteria	Limit State	g_n	\overline{g} (Ref. 5)	\overline{R}/R_n	ϕ	ß (D/L = 0.2)
I and C shapes major axis flexure	3.4.2	Yield Fracture	1.0 1.0	1.07 1.16	1.18 1.28	0.95 0.85	2.9 3.7
I shapes minor axis flexure	3.4.4	Yield Fracture	1.30 1.42	1.30 1.50	1.10 1.16	0.95 0.85	2.5 3.3
Box shapes	3.4.2	Yield Fracture	1.0 1.0	1.10 1.22	1.21 1.34	0.95 0.90	3.0 3.7
Circular tubes	3.4.3	Yield Fracture	1.17 1.24	1.17 1.35	1.10 1.20	0.95 0.85	2.5 3.4
Solid rectangular bars	3.4.4	Yield Fracture	1.30 1.42	1.30 1.50	1.10 1.16	0.95 0.85	2.5 3.3

Table C-3.4-1 DATA FOR TENSION IN EXTREME FIBERS OF BEAMS

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 \overline{B}_T : mean value of the ratio of test results of straight columns to the tangent modulus load. Analysis of the available test results in Reference (3) resulted in the following statistics:

$$\overline{B}_T = 1.0$$
 and $V_{B_T} = 0.05$

This means that the tangent modulus theory is indeed a very good predictor for straight columns.

 \overline{B}_{u} : mean value of the ratio of the ultimate strength of an initially crooked pinned end column to the strength predicted by the tangent modulus theory for straight columns. It was assumed that the initial crookedness of the column is a sine-wave with a maximum amplitude of one-thousandths of the length. This is in accordance with the procedure recommended by the Structural Stability Research Council (Ch. 3, Reference (6)).

The following formulas were derived in Reference (5) for the ratio B_u :

$$\overline{B}_{u} = 1.0 \text{ for } \lambda \leq 0.263
\overline{B}_{u} = 1.05 - 0.19 \lambda \text{ for } 0.263 \leq \lambda \leq 1.20
\overline{B}_{u} = 0.63 + 0.16 \text{ for } 1.20 \leq \lambda \leq 2.0
\overline{B}_{u} = 0.95 \text{ for } \lambda \leq 2.0
V_{B_{u}} = 0.10$$
(30)

A calibration study similar to that presented previously for tension members was performed, using Eq. 1 to determine β , and employing Eqs. 23 and 24 as the nominal column strength: Four different kinds of alloys were investi-

φF₁ (ksi)

gated (Table C-3.4.2). A number of types of relationship for ϕ were tried, and the following expressions were finally selected as being reasonably accurate and yet still fairly simple:

$$\phi_c = 1 - 0.21\lambda \le 0.95 \text{ for } \lambda \le 1.2$$

$$\phi_c = 0.58 + 0.14\lambda \le 0.95 \text{ for } \lambda > 1.2$$

$$(31)$$

The resistance factor thus varies linearly as the slenderness parameter λ . The β values resulting from the use of ϕ_{cc} (Eq. 31) is the LRFD design criteria are shown as the solid curve in Fig. C-5. The target value of $\beta_{T} = 2.5$ is fairly closely approximated.

In Reference (5) considerable work was done on one additional aspect of column design. Real pinned-end columns rarely exist in practice. Even nominally pinned columns have some end restraint, and most columns are actually restrained by the connection to the base or to members framing into their ends. Furthermore, intentionally axially loaded members are also rare, most compression members being actually beamcolumns subjected to both compression and bending. It was shown that each of these effects have a conservative influence and thus they tend to increase β . A number of additional cases were studied, showing the same general trend of a somewhat increased value of β due to restraint.

3.4.8 through 3.4.21

The statistical basis for selecting the ϕ values in these Sections is presented in Reference (3). The same values of ϕ_y were recommended as for tension of the corresponding member types of Sections 3.4.2 through 3.4.4, thus equat-



Figure C-4 COLUMN CURVE FOR 6061-T6 ALLOY

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ing the reliability of short compressed members and elements to that underlying tension elements. The relevant data for choosing the ϕ values, which apply to buckling or crippling type limit states, are summarized in Tables C-3.4-3, C3.4-4, C3.4-5, and C3.4-6. For certain alloys and *Specification* Sections, a negative S_I slenderness limit may result from the equations given in Table 3.4-3. In such cases S_I should be taken as 0.



Table C-3.4-2 DATA USED IN COLUMN CALIBRATION STUDIES

Ref.	Material	Heat Treatment	n	$\sigma_{0.2}$ ksi	E ksi	F _{cy} ksi	V_{R} ***
7	European	No	8	22.78	10,180	20.7*	0.14
8	-	Yes	18.55	40.15	10,100	36.5*	0.14
7	European	Yes	28.60	43.99	10,790	40.0*	0.14
9	6061-T6	Yes	15.5	40.8	10,100	35**	0.14

* $F_{cy} = \sigma_{0.2}/1.1$, assuming $\sigma_{0.2}$ to be the mean yield stress

** Specified value

*** $V_R = \sqrt{0.05^2 + 0.06^2 + 0.05^2 + 0.10^2} = \sqrt{V_A^2 + V_{\sigma_{TM}}^2 + V_{B_T}^2 + V_{B_H}^2}$

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Table C-3.4-3 SUMMARY OF STATISTICAL DATA

Sec. in Ref. 1	Limit State	F.S.	P_m	M_m	F_m	$\frac{R_m}{R_n}$	V_P	V_M	V_F	V_R	Category
3.4.1, 2,	Y	n_{y}	1.0	1.10	1.0	1.10	0	0.06	0.05	0.08	А
3, 4	U	$k_t n_u$	1.0	1.10	1.0	1.10	0	0.06	0.05	0.08	В
3.4.8, 9	Y	n_{y}	1.0	1.10	1.0	1.10	0	0.06	0.05	0.08	С
	В	n_u	1.0	1.0	1.0	1.0	0.05	0.06	0.05	0.09	D
3.4.10	Y	n_{y}	1.0	1.10	1.0	1.10	0	0.06	0.05	0.08	С
	IB	n_u	1.0	1.0	1.0	1.0	0.05	0.06	0.05	0.09	D
	EB	n_u	1.24	1.0	1.0	1.24	0.27	0.06	0.05	0.28	Е
3.4.11,	Y	n_{y}	1.0	1.10	1.0	1.10	0	0.06	0.05	0.08	А
13, 14	В	n_y	1.03	1.0	1.0	1.03	0.11	0.06	0.05	0.13	F
3.4.12,	Y	n_{y}	1.0	1.10	1.0	1.10	0	0.06	0.05	0.08	А
16.1	IB	n_y	1.01	1.0	1.0	1.01	0.05	0.06	0.05	0.09	G
	EB	n_y	1.24	1.0	1.0	1.24	0.27	0.06	0.05	0.28	Н
3.4.15,	Y	n_{v}	1.0	1.10	1.0	1.10	0	0.06	0.05	0.08	А
16, 17	В	n_{y}	1.0	1.0	1.0	1.0	0.05	0.06	0.05	0.09	Ι
3.4.20	Y	$n_{\rm v}$	1.0	1.10	1.0	1.10	0	0.06	0.05	0.08	А
	IB	n_{y}	1.07	1.0	1.0	1.07	0.09	0.06	0.05	0.12	J
	EB	n_y	0.93	1.0	1.0	0.93	0.09	0.06	0.05	0.12	K

Table C-3.4-4 LIMIT STATE CATEGORIES

Category FS \overline{R}/R_n V_R Description

Table C-3.4-5 RELIABILITY INDICES FOR ASD SPECIFICATION

				_			
А	1.65	1.10	0.08	yield in tension	Category	β	β
В	1.95	1.10	0.08	fracture in tension		for $D/L = 0.1$	for $D/L = 0.2$
С	1.65	1.10	0.08	yield in compression	А	2.46	2.64
D	1.95	1.00	0.09	buckling of column	В	3.16	3.40
				components	С	2.87	3.09
				inelastic column buckling	D	2.72	2.92
Е	1.95	1.24	0.28	elastic column buckling	Е	2.44	2.51
F	1.65	1.03	0.13	beam buckling, overall	F	2.01	2.13
G	1.65	1.01	0.09	inelastic local buckling	G	2.08	2.22
Н	1.65	1.24	0.28	elastic local buckling	Н	1.98	2.03
Ι	1.65	1.00	0.09	local buckling of beams	I	2.04	2.18
J	1.65	1.07	0.12	inelastic shear buckling	I	2.04	2.10
K	1.65	0.93	0.12	elastic shear buckling	J	1.65	1.75
					К	1.05	1./5